
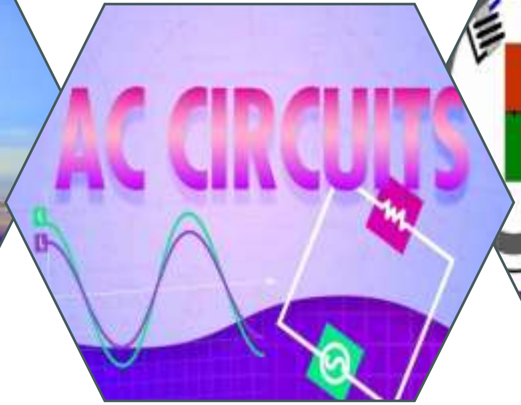

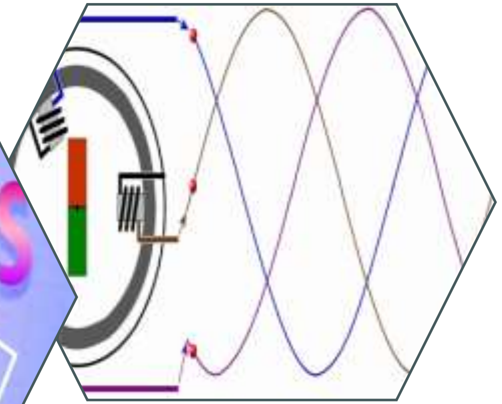

Electrical Circuit- II

Prepared by :
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University of Global Village (UGV), Barishal

Basic Course Information

Course Title	Electrical Circuit -II
Course Code	EEE 0713-1102
Credit	03
Marks	150

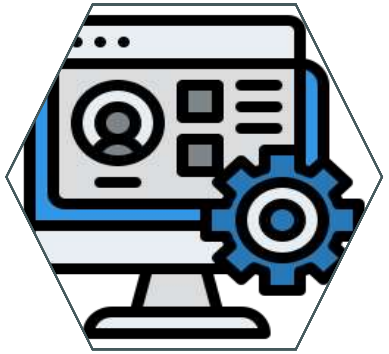
SYNOPSIS/RATIONALE

This course is offered to the students of the EEE department to develop fundamental concepts of transient analysis of electrical circuits, coupled and poly-phase circuits and resonance analysis. Students will also learn the concepts of balanced and unbalanced circuits.

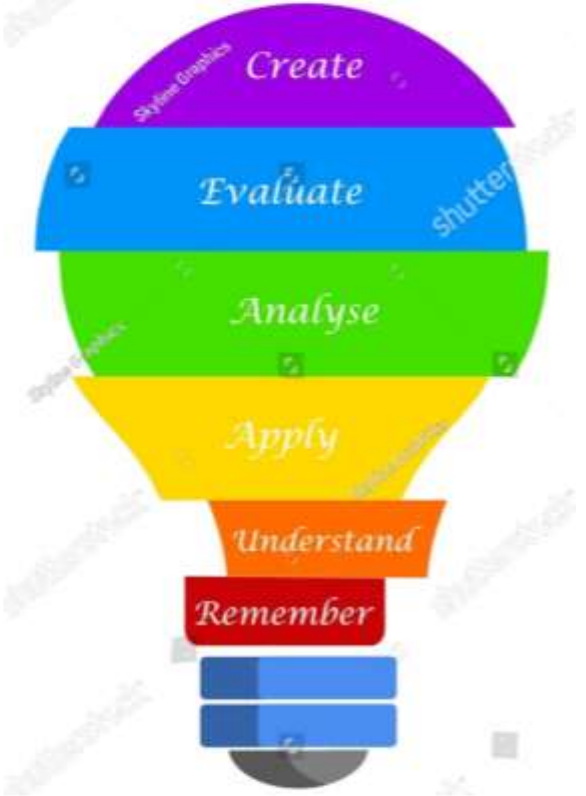
OBJECTIVE

- This course has been designed for the students.
- To familiarize with the transient condition analysis of electrical (AC & DC) circuits
- To provide knowledge about single and three-phase AC circuits
- To provide analytical ideas about resonance in AC circuits
- To evaluate a balanced and unbalanced AC system

Course Learning Outcome (CLO)



CLO-1:	**CLO-2:**	**CLO-3:**	**CLO-4:**
Describe the transient and resonance conditions in simple electrical circuits	**Apply different techniques to solve AC circuits in the phasor domain**	**Evaluate resonance circuit characteristics used in ac power transfer**	**Design different types of balanced and unbalanced systems.**



ASSESSMENT PATTERN

CIE- Continuous Internal Evaluation (90 Marks)

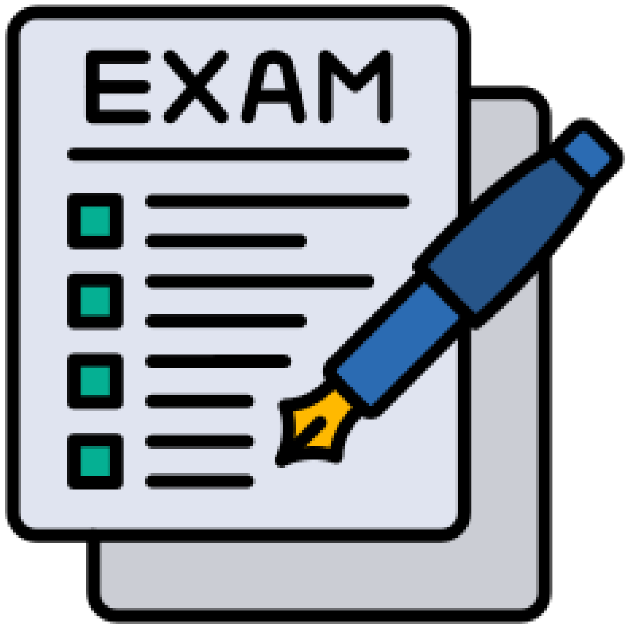
Bloom's Category Marks (out of 90)	**Tests Mid-term (45)**
Remember	08
Understand	08
Apply	08
Analyze	08
Evaluate	08
Create	05

Class Test	**15**
Presentation	**15**
Attendance	**15**

ASSESSMENT PATTERN

SEE- Semester End Examination (60 Marks)

Bloom's Category	Tests
Remember	**10**
Understand	**10**
Apply	**10**
Analyze	**10**
Evaluate	**10**
Create	**10**



Assessment Strategy Overview

Quizzes:	Assignments	Midterm Exam	Mock Test	Final Exam
Weeks 1, 3, 5, 7, 11, 12, and 14 to evaluate theoretical knowledge.	Weeks 2, 6, 10, 13, and 15 to practice problem-solving skills.	Week 8, covering content from Weeks 1–7.	Week 16 for final exam preparation.	Week 17, comprehensive assessment across all CLOs.

COURSE CONTENT

- ✓ Basic characteristics of sinusoidal functions.
- ✓ Forced response of first-order circuits to sinusoidal excitation.
- ✓ Instantaneous, average, and reactive power due to sinusoidal excitation, effective values, and power factor.
- ✓ Complex exponential forcing functions, phasors, impedance, and admittance.
- ✓ Basic circuit laws for AC circuits.
- ✓ Nodal and mesh analysis, network theorems for AC circuits.
- ✓ Balanced and unbalanced three-phase circuits, power calculation.

Time distributions

Course Content	CLOs	Hours
Basic characteristics of sinusoidal functions	CLO1	4
Forced response of first-order circuits to sinusoidal excitation	CLO2	4
Instantaneous, average, and reactive power due to sinusoidal excitation, effective values, and power factor	CLO3	6
Complex exponential forcing functions, phasors, impedance, and admittance	CLO1, CLO2	6
Basic circuit laws for AC circuits	CLO1	4
Nodal and mesh analysis, network theorems for AC circuits	CLO2, CLO3	6
Balanced and unbalanced three-phase circuits, power calculation	CLO3, CLO4	4

Course Schedule

Week	Course Content	Teaching-Learning Strategy	Sources	Assessment Strategy	CLOs
1	Basic characteristics of sinusoidal functions	Lecture, examples, Q&A	Khan Academy: Sinusoidal Functions & Slide	Class Quiz on sinusoidal properties	CLO1
2	Sinusoidal excitation and forced response of first-order circuits	Lecture with worked examples	Neso Academy: First-Order Circuits & Slide	Assignment 1: Forced response problems	CLO2
3	Instantaneous, average, and RMS values in sinusoidal excitation	Lecture, problem-solving	All About Circuits: Power Calculations & Slide	Quiz: Power calculations	CLO3
4	Complex exponential forcing functions and phasors	Lecture, interactive discussion	MIT OCW: Phasor Analysis & Slide	Problem-solving exercise	CLO3
5	Different types of power and power factors for sinusoidal functions	Lecture and discussions	YouTube: Power Factor & Slide	Class Quiz on phasors	CLO1, CLO2

Course Schedule(Cont.)

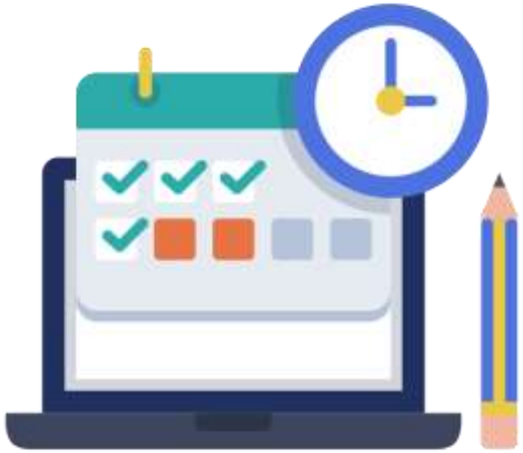
Week	**Course Content**	**Teaching-Learning Strategy**	**Sources**	**Assessment Strategy**	**CLOs**
6	Impedance and admittance in AC circuits	Lecture, problem-solving sessions	YouTube: Impedance Basics	Assignment 2: Impedance problems	CLO1, CLO2
7	Basic circuit laws for AC circuits: KVL and KCL	Lecture, problem-solving	KVL/KCL Tutorial	Quiz: KVL/KCL applications	CLO1
8	Nodal analysis in AC circuits	Lecture, examples, group discussions	Nodal Analysis	Midterm Exam covering Weeks 1–7	CLO2
9	Mesh analysis in AC circuits	Lecture, step-by-step examples	Mesh Analysis	Problem-solving exercise	CLO2
10	Network theorems: Thevenin and Norton's Theorem	Lecture with practical examples	Thevenin/Norton Theorem	Assignment 3: Thevenin/Norton problems	CLO2, CLO3

Course Schedule (Cont.)

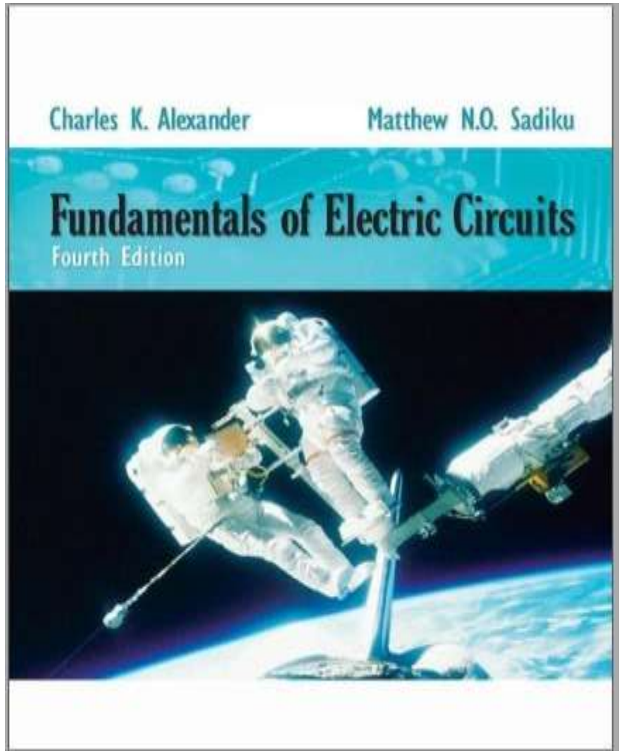
Week	Course Content	Teaching-Learning Strategy	Sources	Assessment Strategy	CLOs
11	Superposition Theorem and applications	Lecture, problem-solving	Superposition Theorem	Quiz: Network theorems	CLO2, CLO3
12	Balanced three-phase circuits: Basics	Lecture, problem-solving	Three-Phase Circuits	Class Quiz on balanced circuits	CLO3, CLO4
13	Power calculations in balanced three-phase systems	Lecture, step-by-step calculations	Balanced Power Calculation	Assignment 4: Power calculations	CLO3, CLO4
14	Unbalanced three-phase circuits: Theory and analysis	Lecture, examples	Unbalanced Circuits	Quiz: Unbalanced circuit problems	CLO3, CLO4
15	Power factor correction techniques in three-phase systems	Lecture, Q&A, real-world examples	Power Factor Correction	Group Assignment: Power factor solutions	CLO3, CLO4

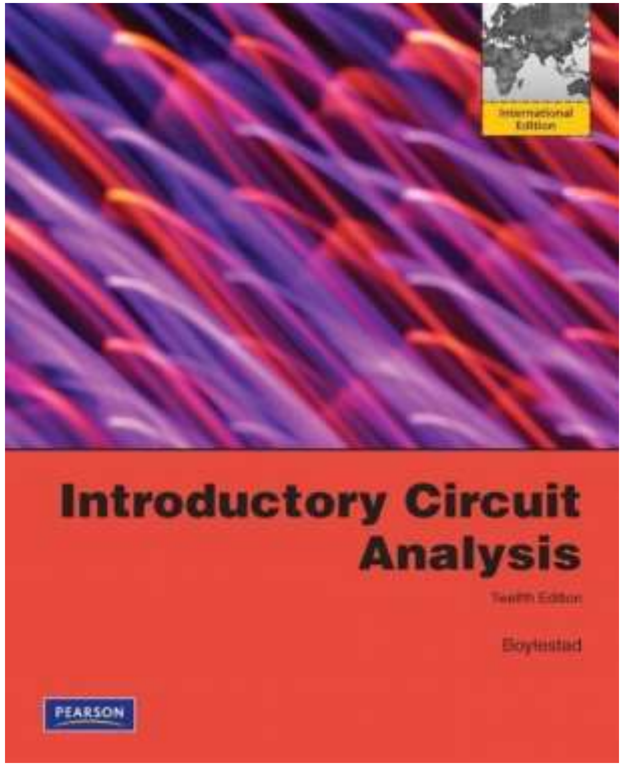
Course Schedule (Cont.)

Week	**Course Content**	**Teaching-Learning Strategy**	**Sources**	**Assessment Strategy**	**CLOs**
16	Review of all key topics and advanced problem-solving	Revision lectures, practice questions	Course slides and previous sources	Mock Test for final exam preparation	CLO1–CLO4
17	Final assessment and student feedback	Comprehensive review and Q&A	Course slides and review material	Final Exam: Comprehensive	CLO1–CLO4



Reference Books





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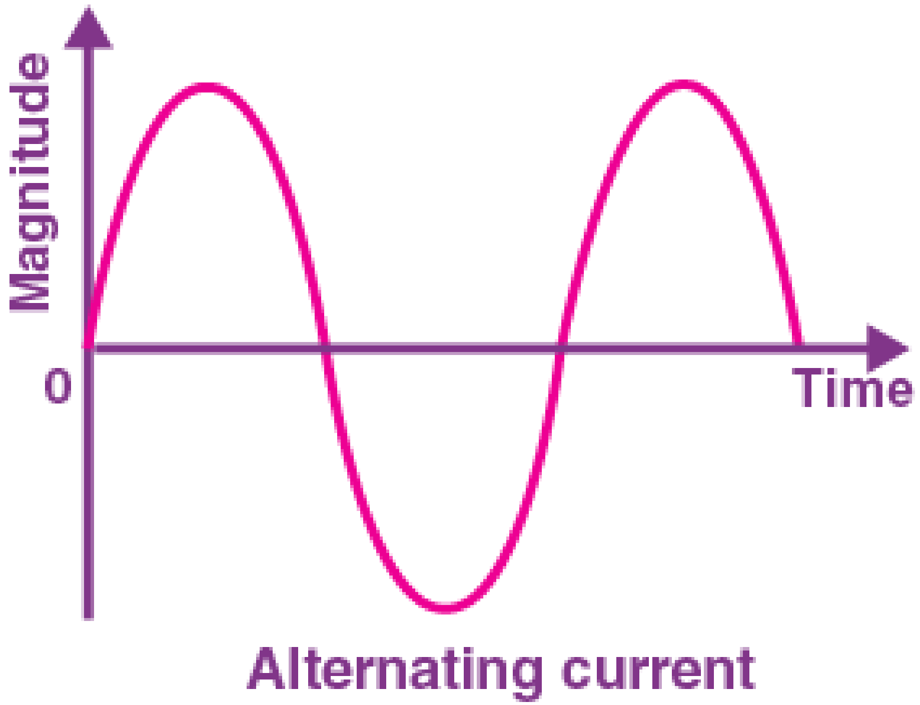
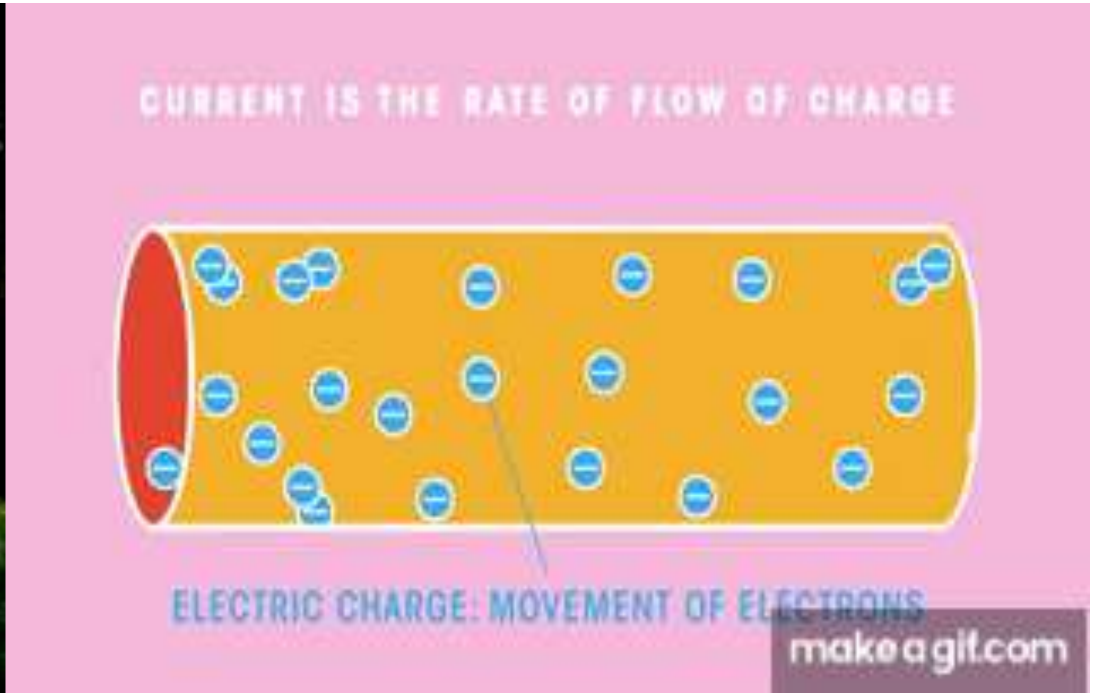
Basic characteristics of Sinusoidal Functions



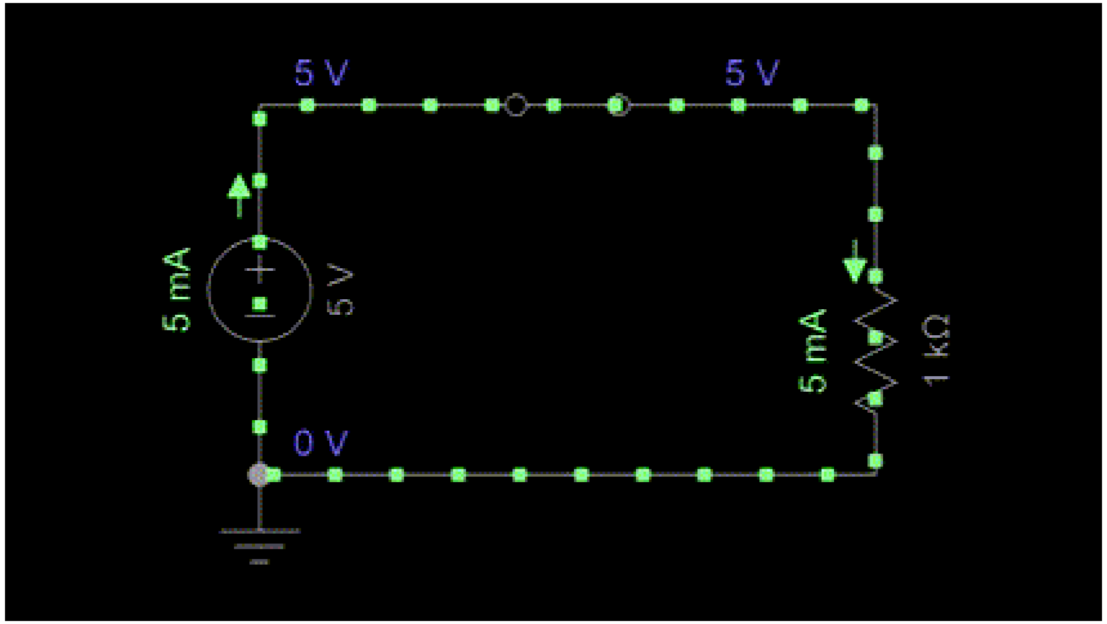
Introduction to Current

Current: Flow of Water.

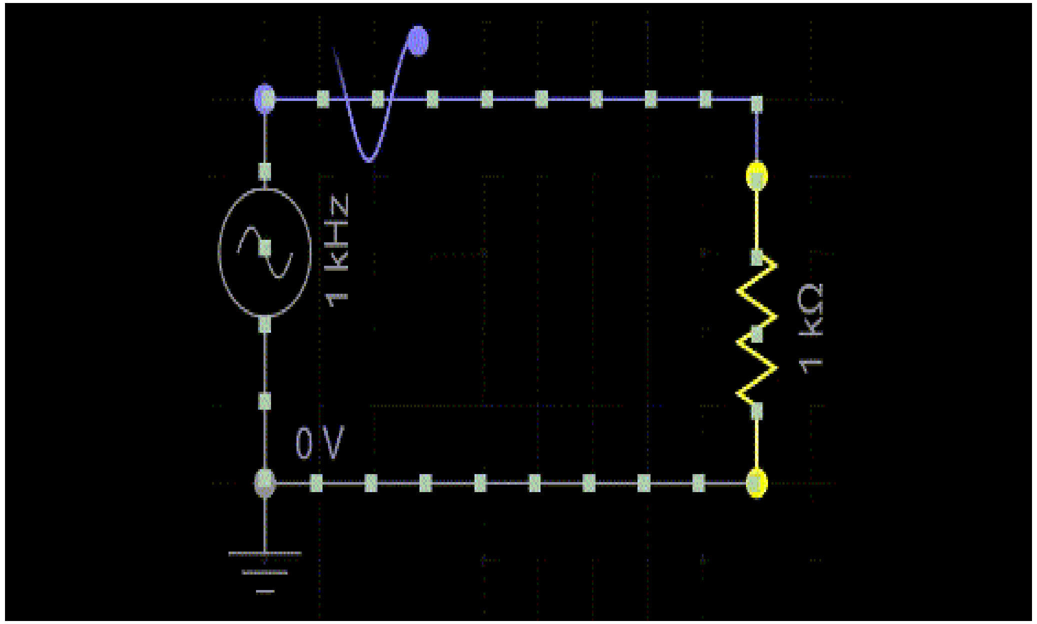


Electric Current: Flow of Electron.

Types of Electric Current



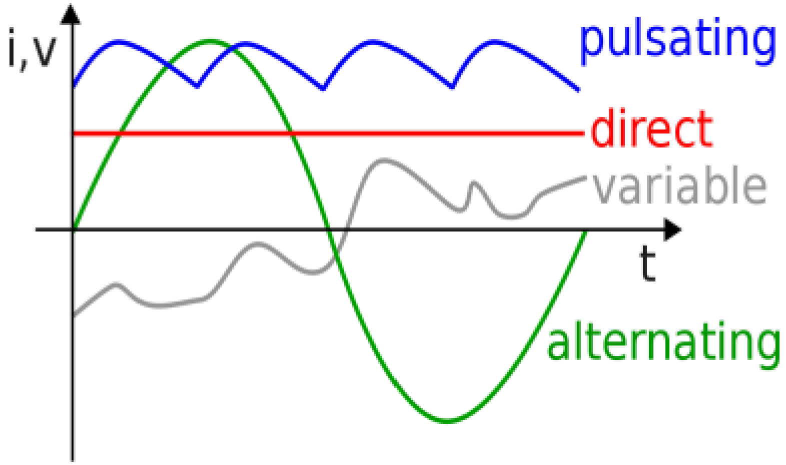
Direct Current (DC)



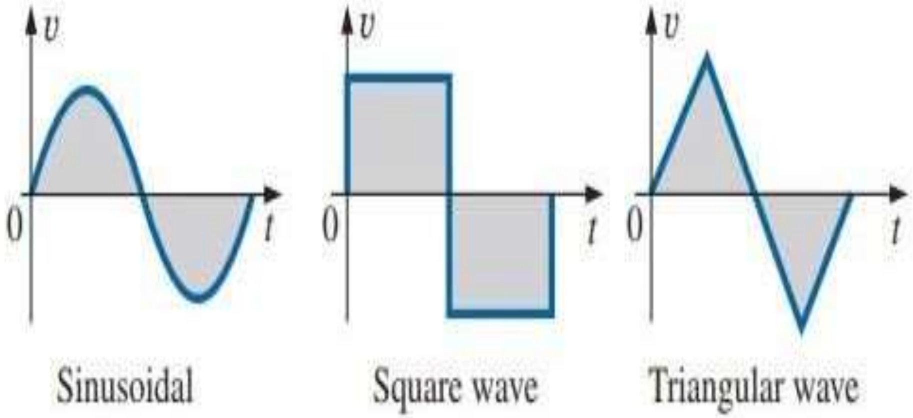
Alternating Current (AC)

What is AC?

AC stands for 'alternating current' which means the current constantly changes direction.

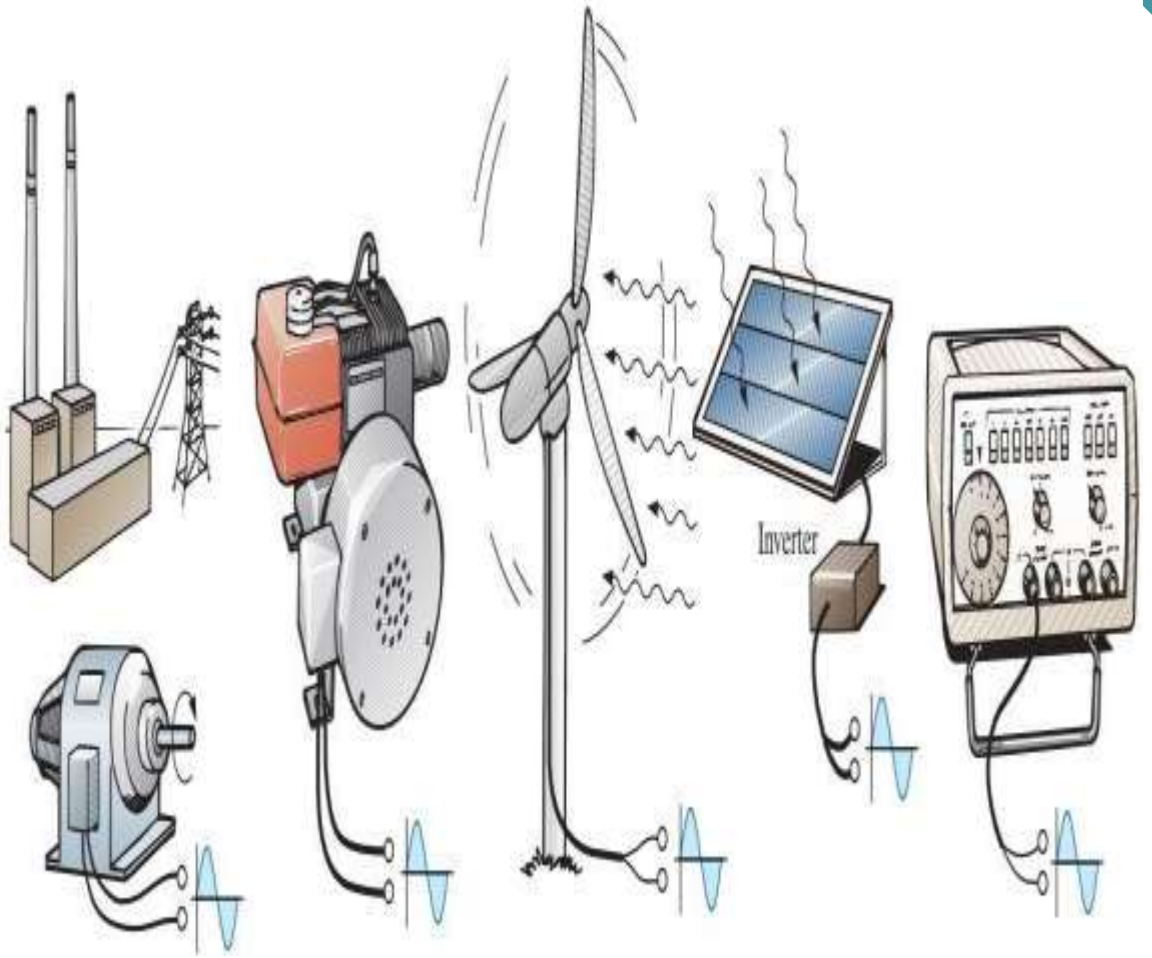


Alternating Current (AC)



How AC voltage can be generated?

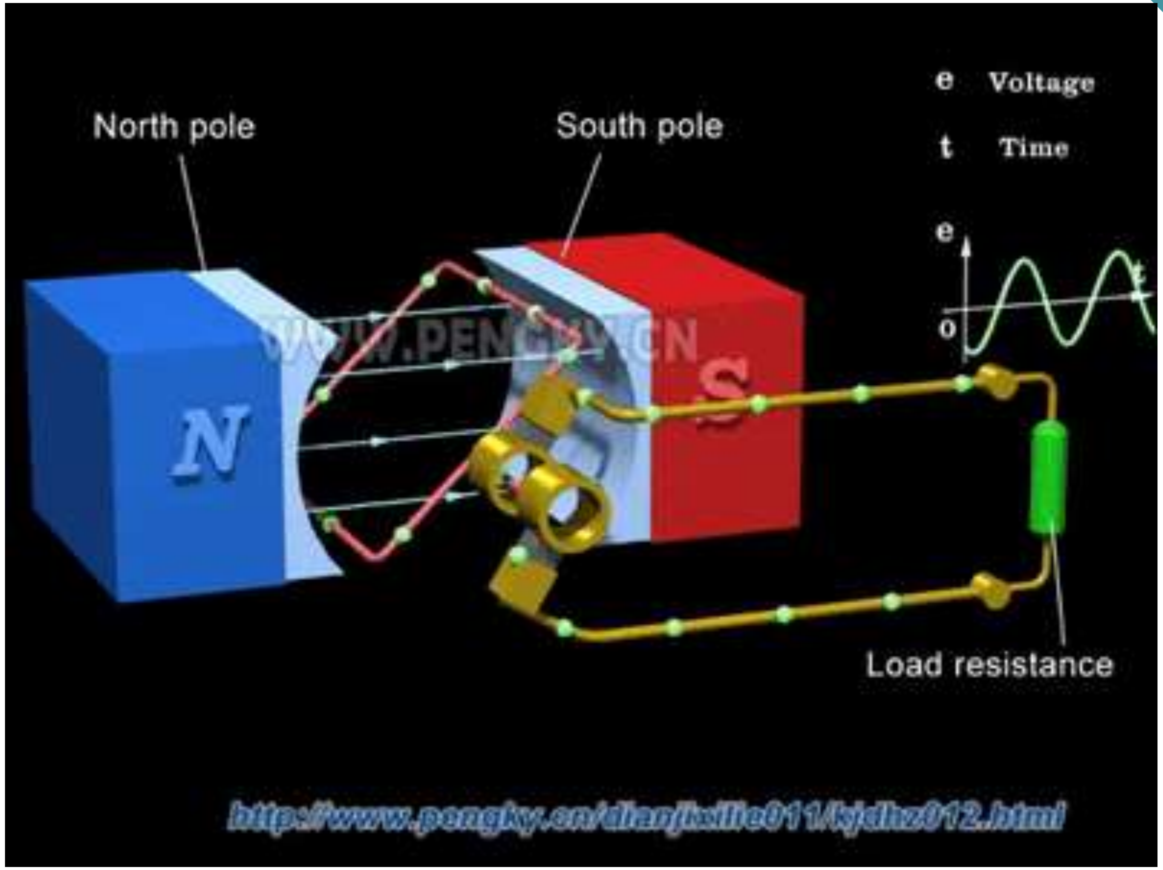
Sinusoidal ac voltages are available from a variety of sources. The most common source is the typical home outlet, which provides an ac voltage that originates at a power plant. Most power plants are fueled by water power, oil, gas, or nuclear fusion.



How AC voltage can be generated?

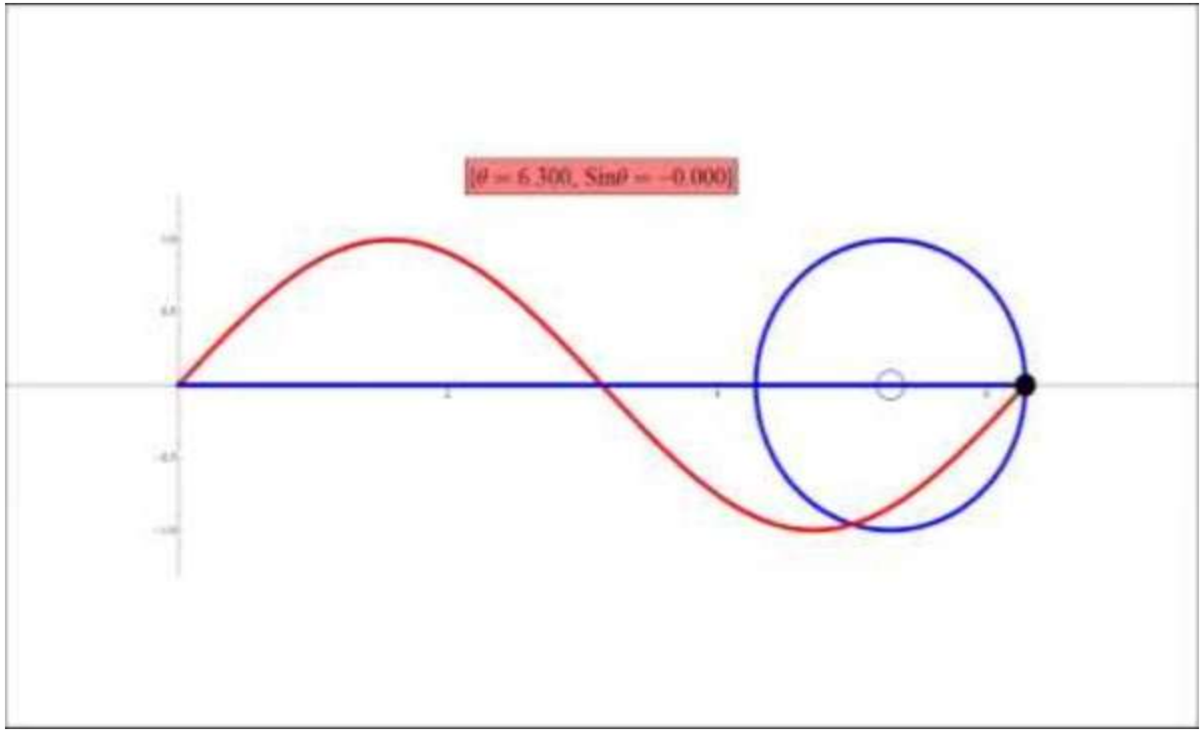
In each case, an AC generator/alternator as shown in Fig. is the primary component in the energy-conversion process. The power to the shaft developed by one of the energy sources listed turns a rotor (constructed of alternating magnetic poles) inside a set of windings housed in the stator (the stationary part of the dynamo) and induces a voltage across the windings of the stator, as defined by Faraday's law,

$$E = N\frac{d\varphi}{dt}$$



Visualization

How the Sinusoidal Function is related to the rotation of the Armature!



Characteristic	AC Power	DC Power
Direction of Flow	Alternates direction	Constant, one direction
Typical Uses	Home and office electricity, large motors	Electronic devices, batteries
Transmission Distance	Long-distance(with less energy loss)	Short-distance (better for energy storage)
Conversion	Transformable voltage levels	Direct and stable output

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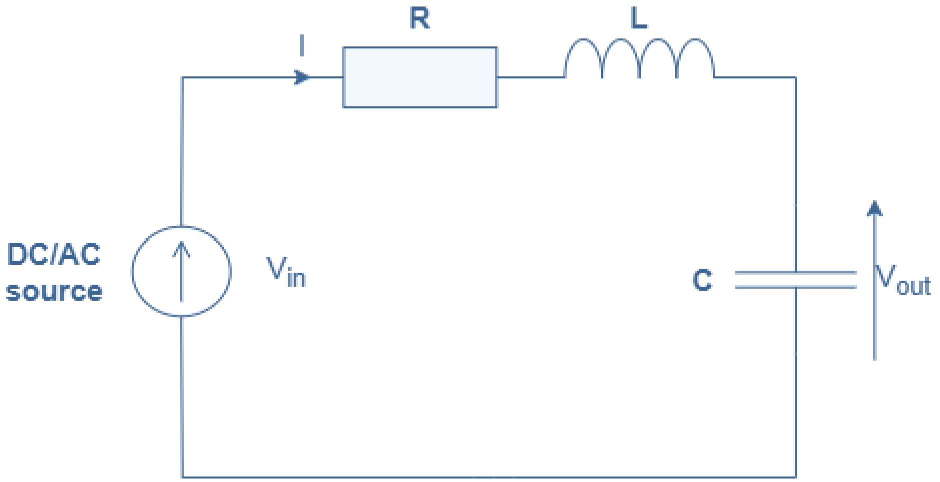
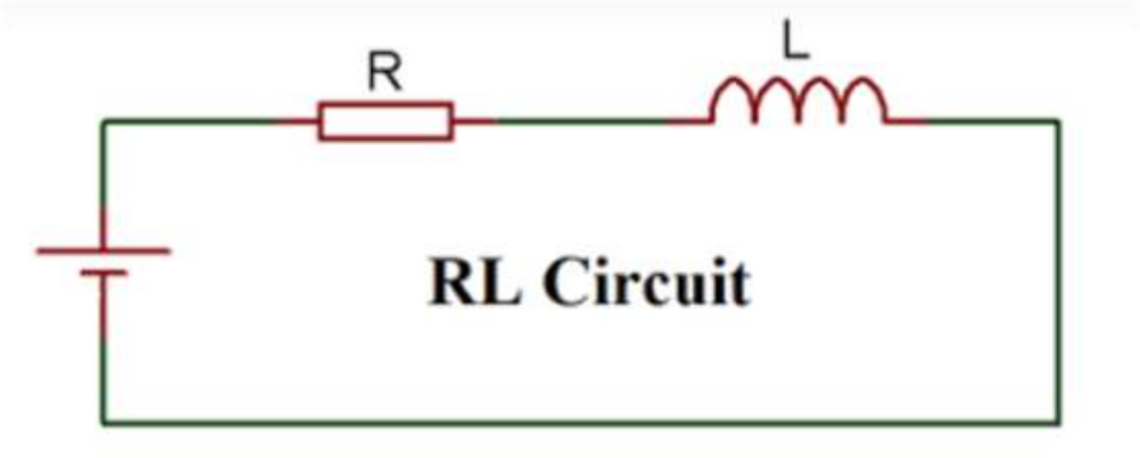
Sinusoidal excitation and forced response of first-order circuits



Introduction

First-order circuits:

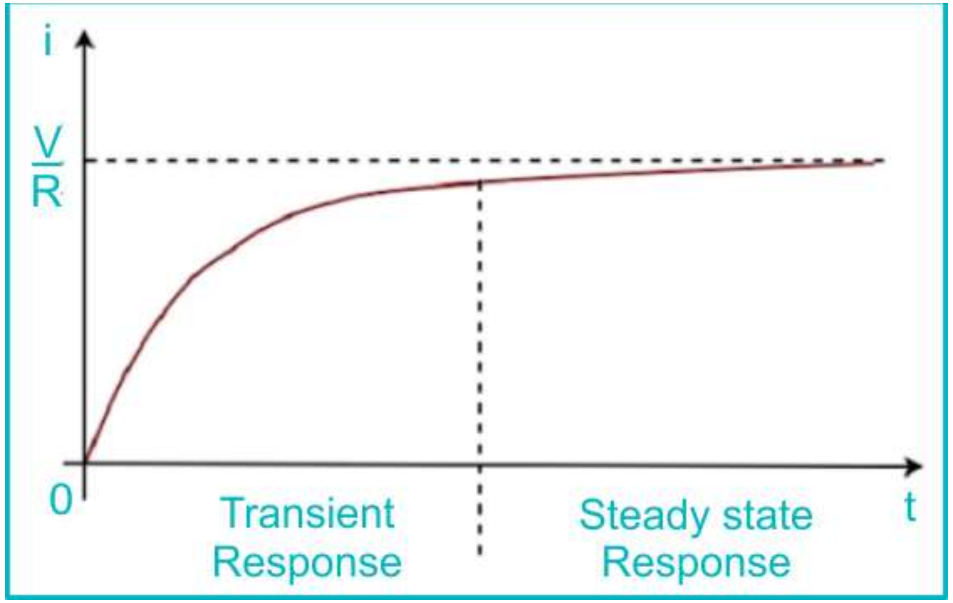
Circuits containing one energy storage element (capacitor or inductor) and resistors.



Introduction (Cont.)

Why sinusoidal excitation?

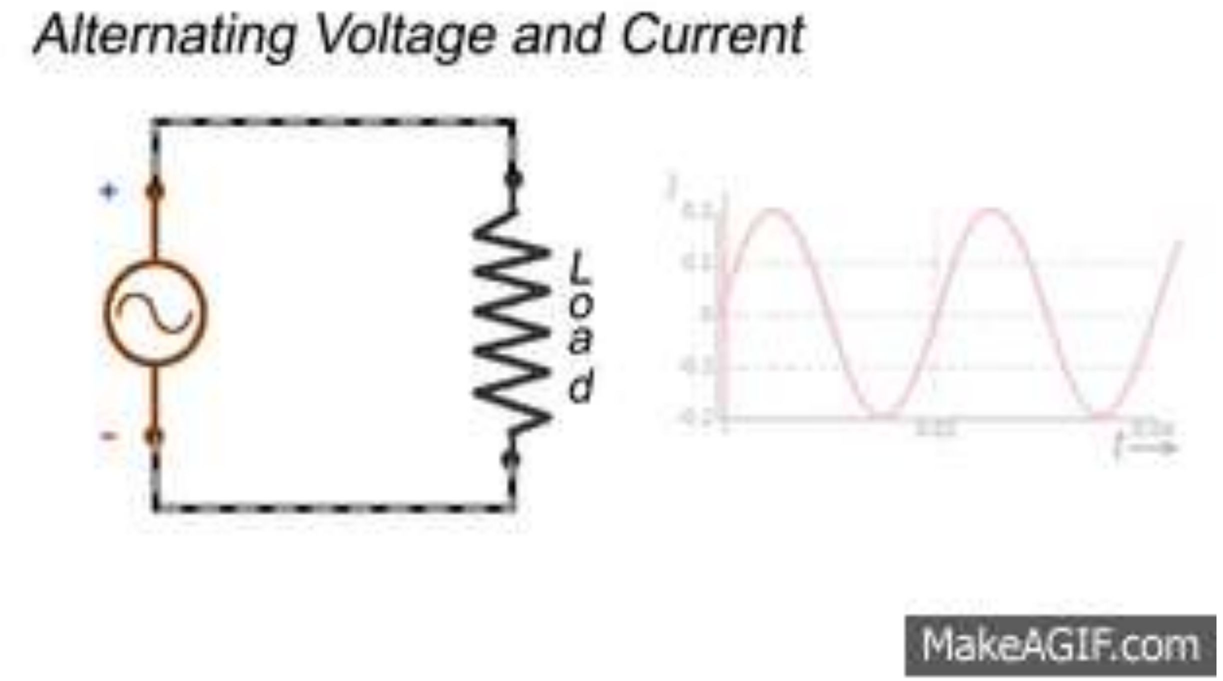
Sinusoidal signals are foundational in AC analysis due to their steady-state behavior and periodic nature



Applications: Used in filters, oscillators, and communication organisms.

AC Excitation with Resistive Load

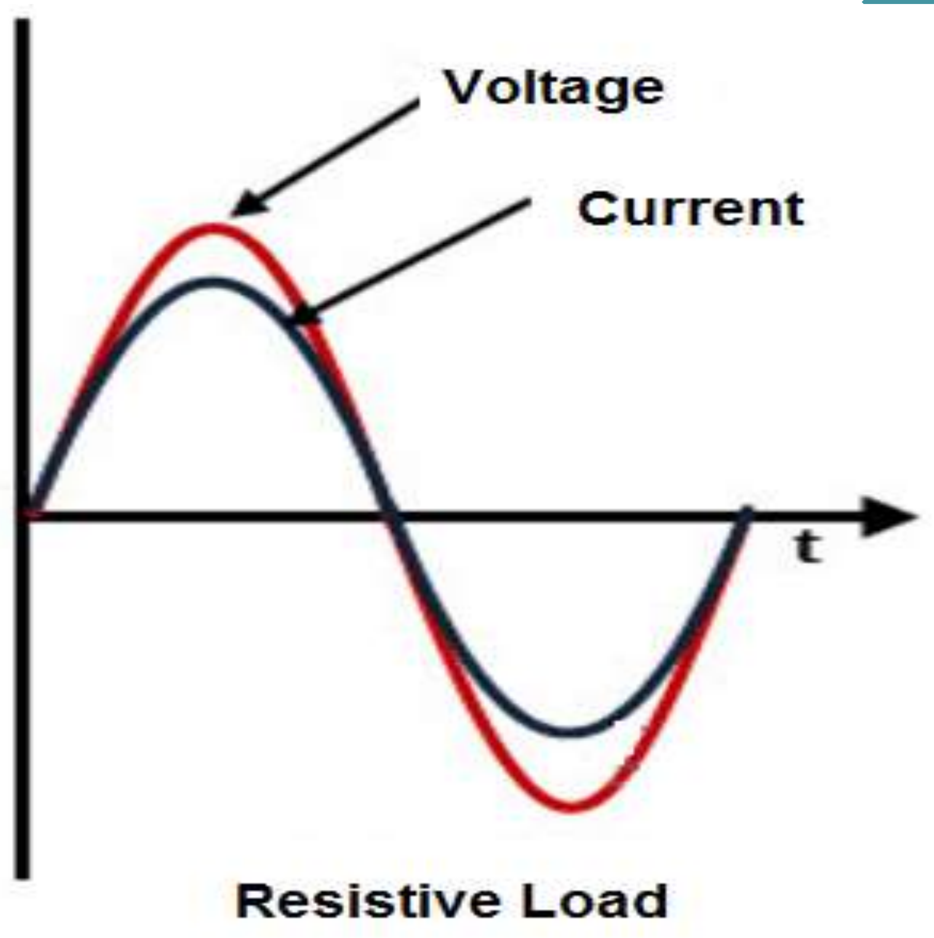
Resistive Load Behavior: When an AC voltage source is connected to a purely resistive load, the current and voltage are in phase.



AC Excitation with Resistive Load (Cont.)

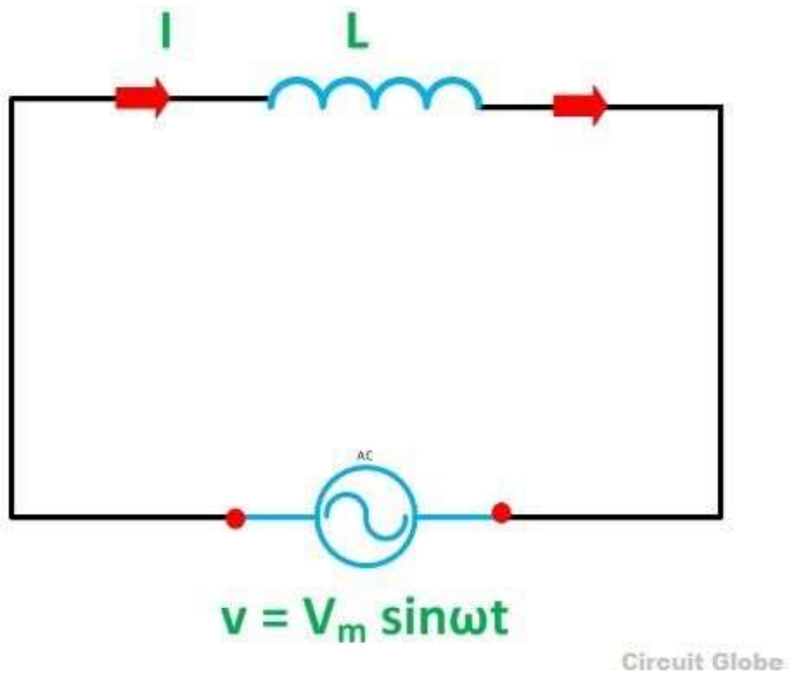
Voltage and current have the same sinusoidal shape, and there is no phase shift (ϕ=0).

Practical Examples: Electric heaters, incandescent bulbs, and other resistive devices.



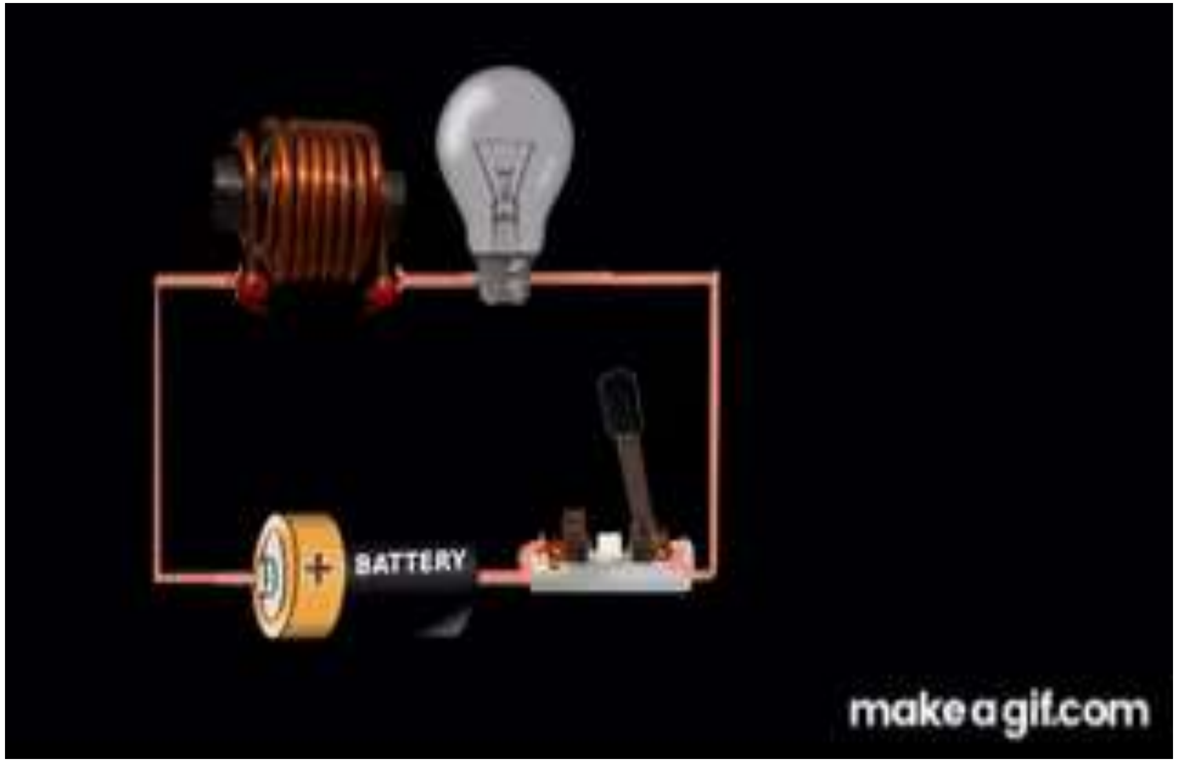
AC Excitation with Inductive Load

Inductive Load Behavior: When an AC voltage source is connected to an inductor, the current lags the voltage by 90ϕ=90).



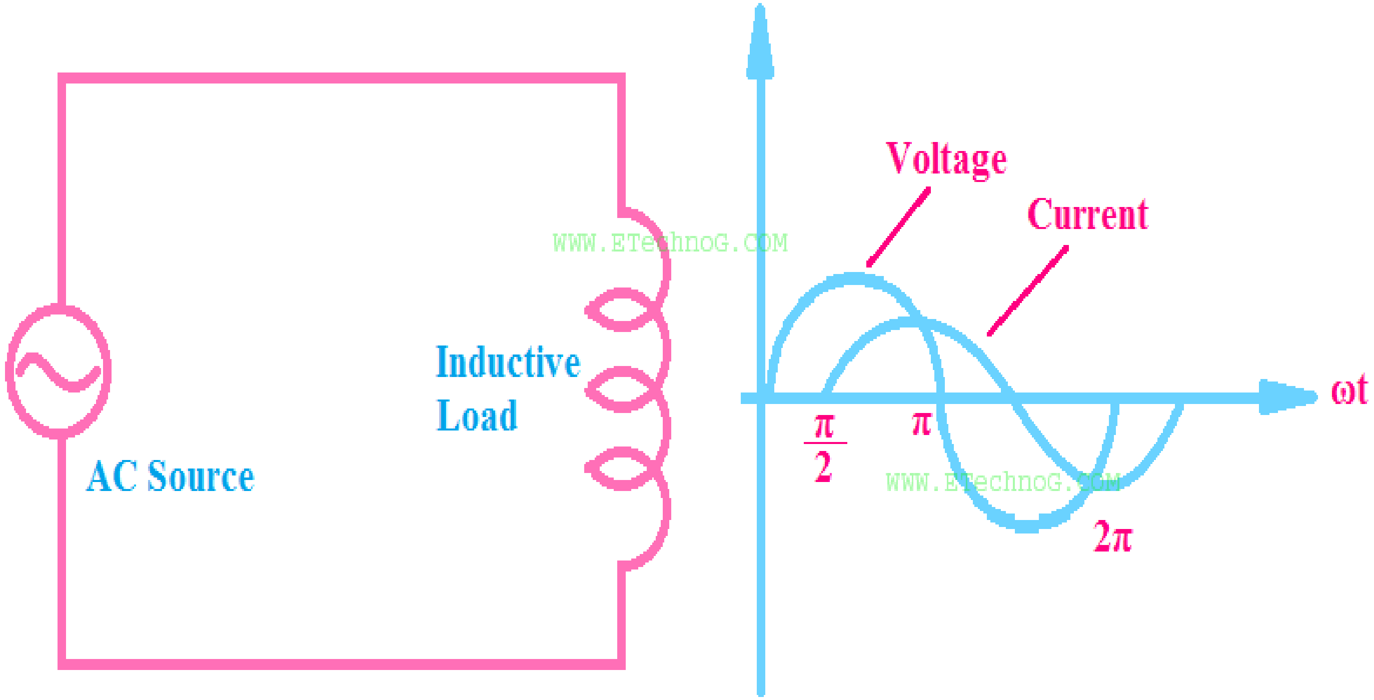
AC Excitation with Inductive Load (Cont.)

This lag occurs because the inductor opposes changes in current, storing energy in its magnetic field.



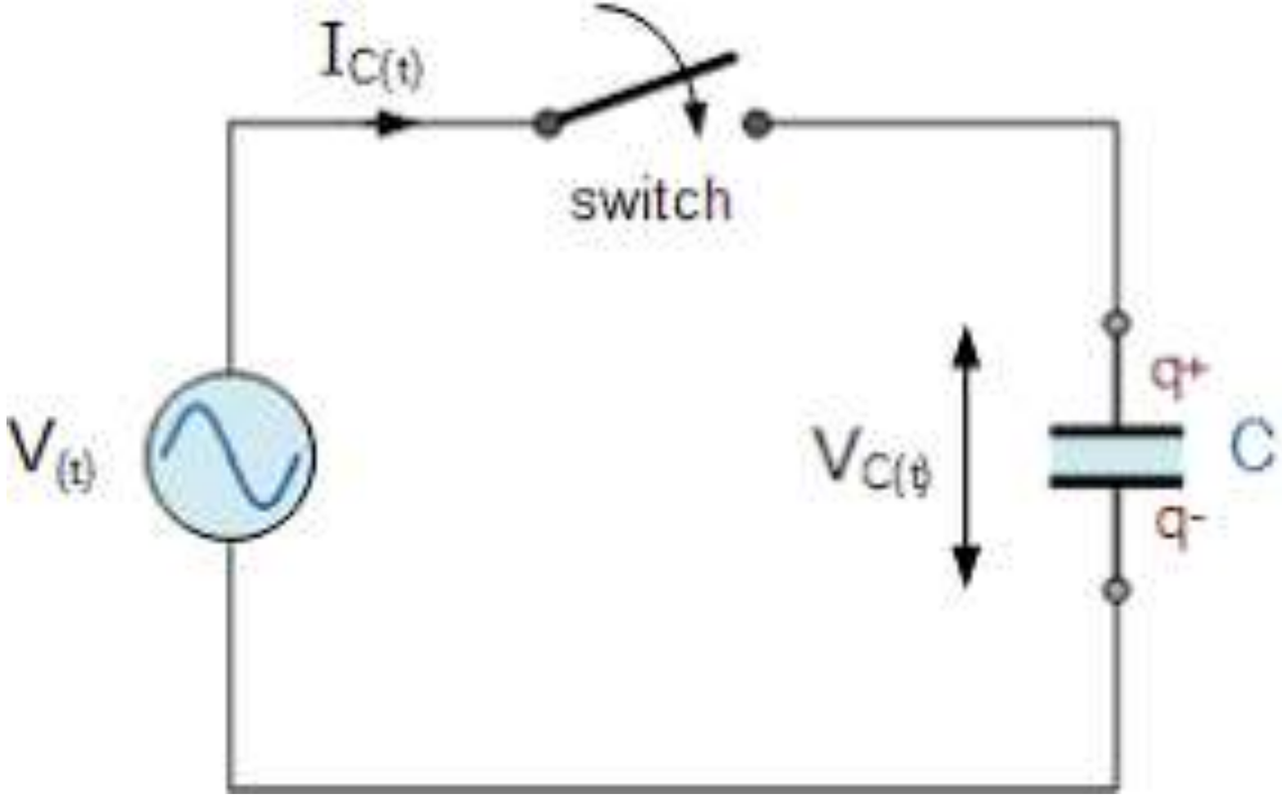
AC Excitation with Inductive Load (Cont.)

Inductive Reactance: Opposition offered by an inductor to AC: X_L==ωL, where $\omega=2\pi f$ is the angular frequency. The higher the frequency, the greater the inductive reactance.



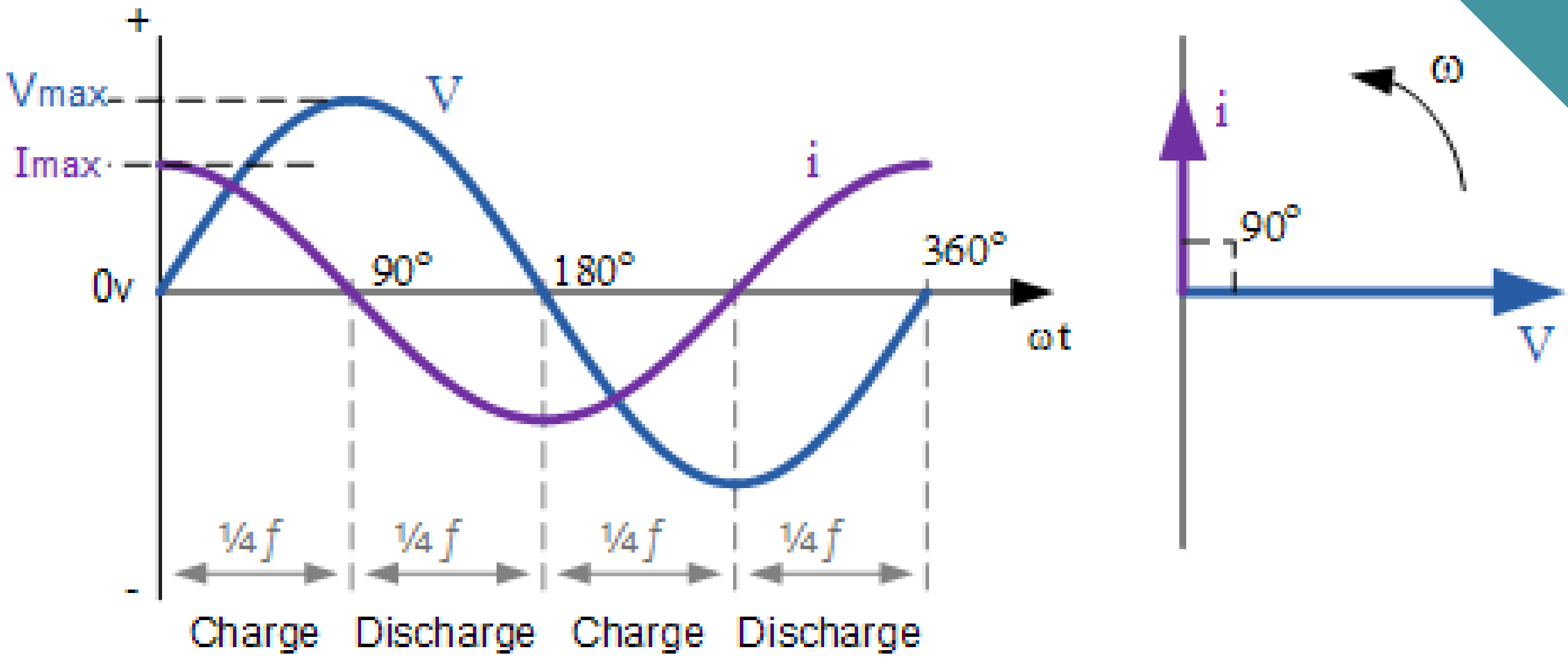
AC Excitation with Capacitive Load

Capacitive Load Behavior: When an AC voltage source is connected to a capacitor, the current leads the voltage by 90. This phase shift occurs because the capacitor stores energy in the electric field and opposes voltage changes.



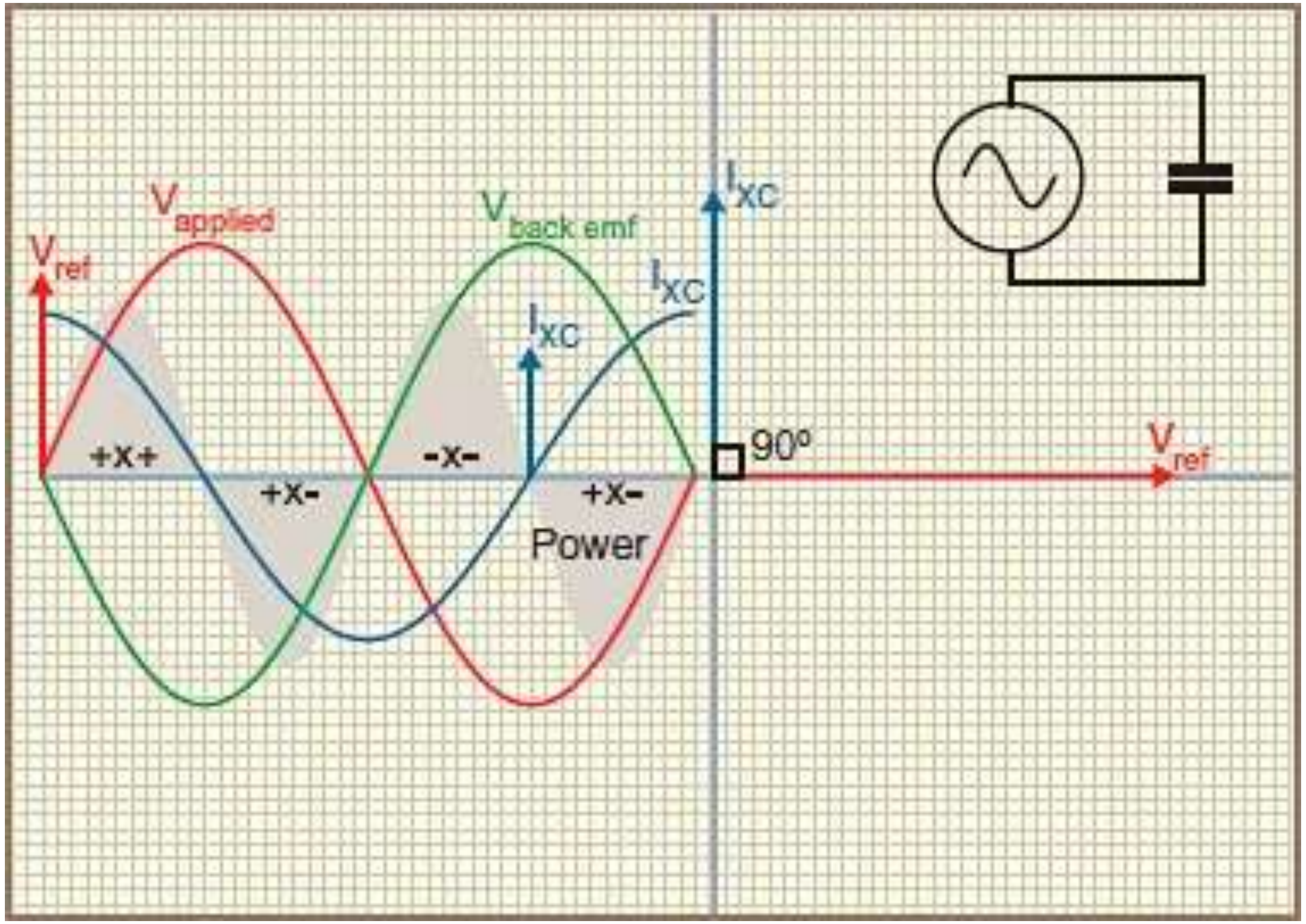
AC Excitation with Capacitive Load (Cont.)

Capacitive Reactance: Opposition offered by a capacitor to AC is given by $X_C = 1/\omega C$, where $\omega = 2\pi f$. The lower the frequency, the higher the capacitive reactance.

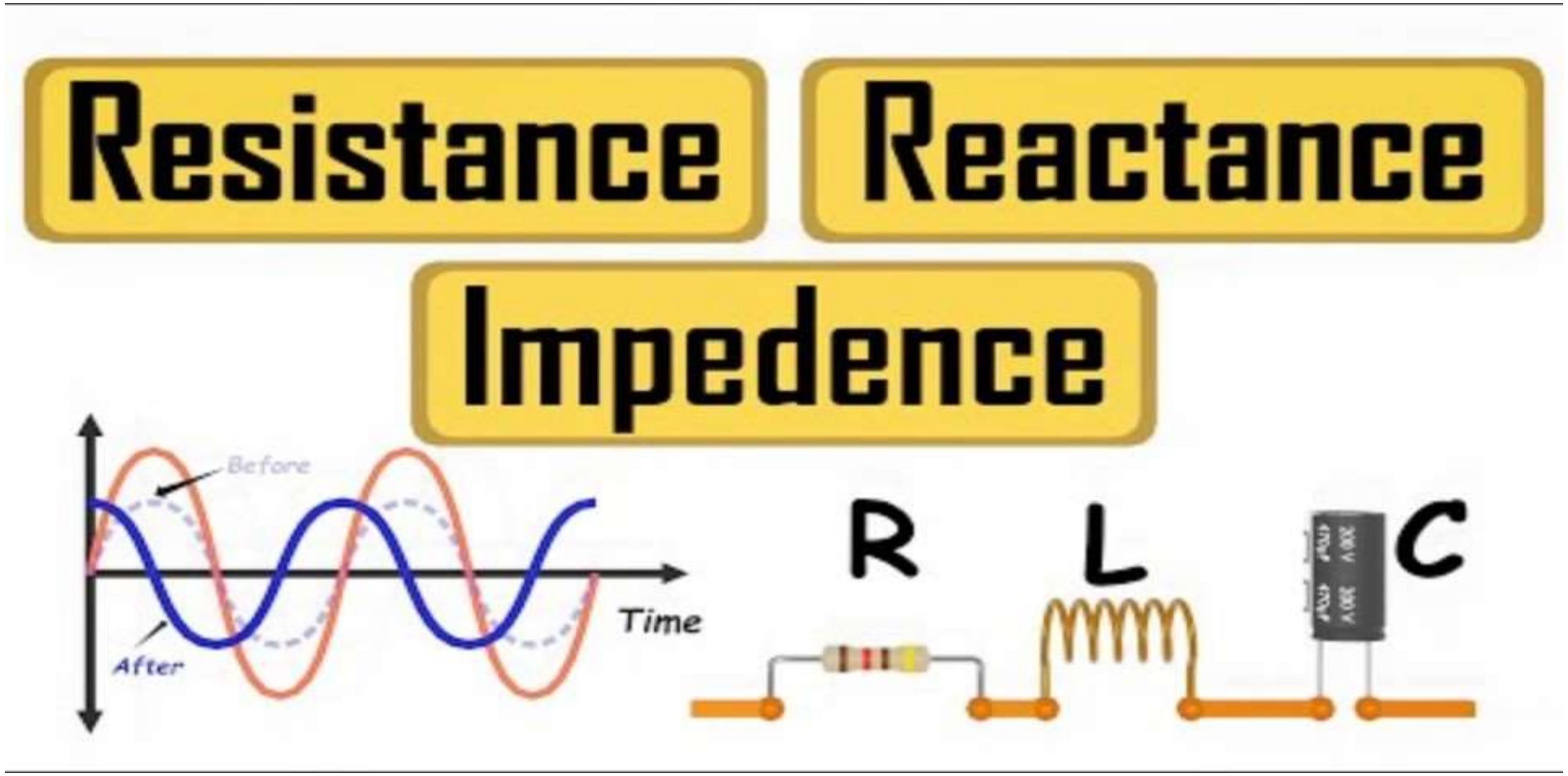


AC Excitation with Capacitive Load (Cont.)

Power in Capacitive Loads: The power alternates between positive and negative, and the average power is zero (purely reactive power). Capacitors do not dissipate energy but transfer it back and forth with the source



Resistance, Reactance & Impedance



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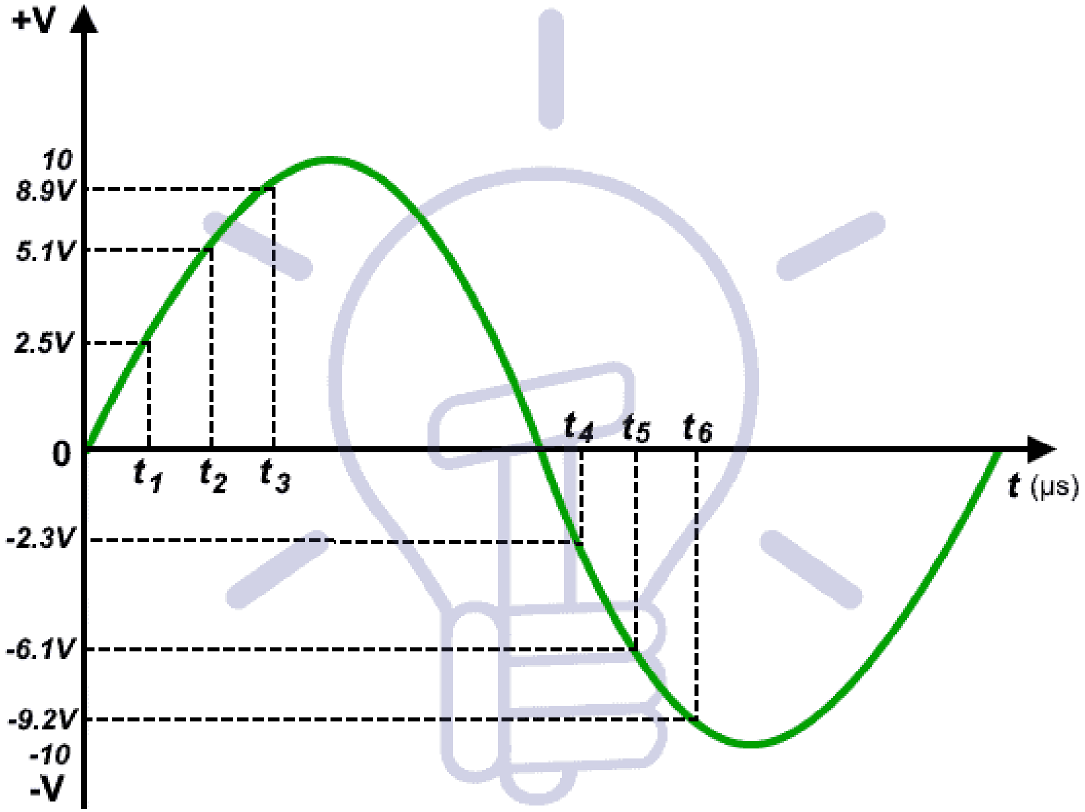
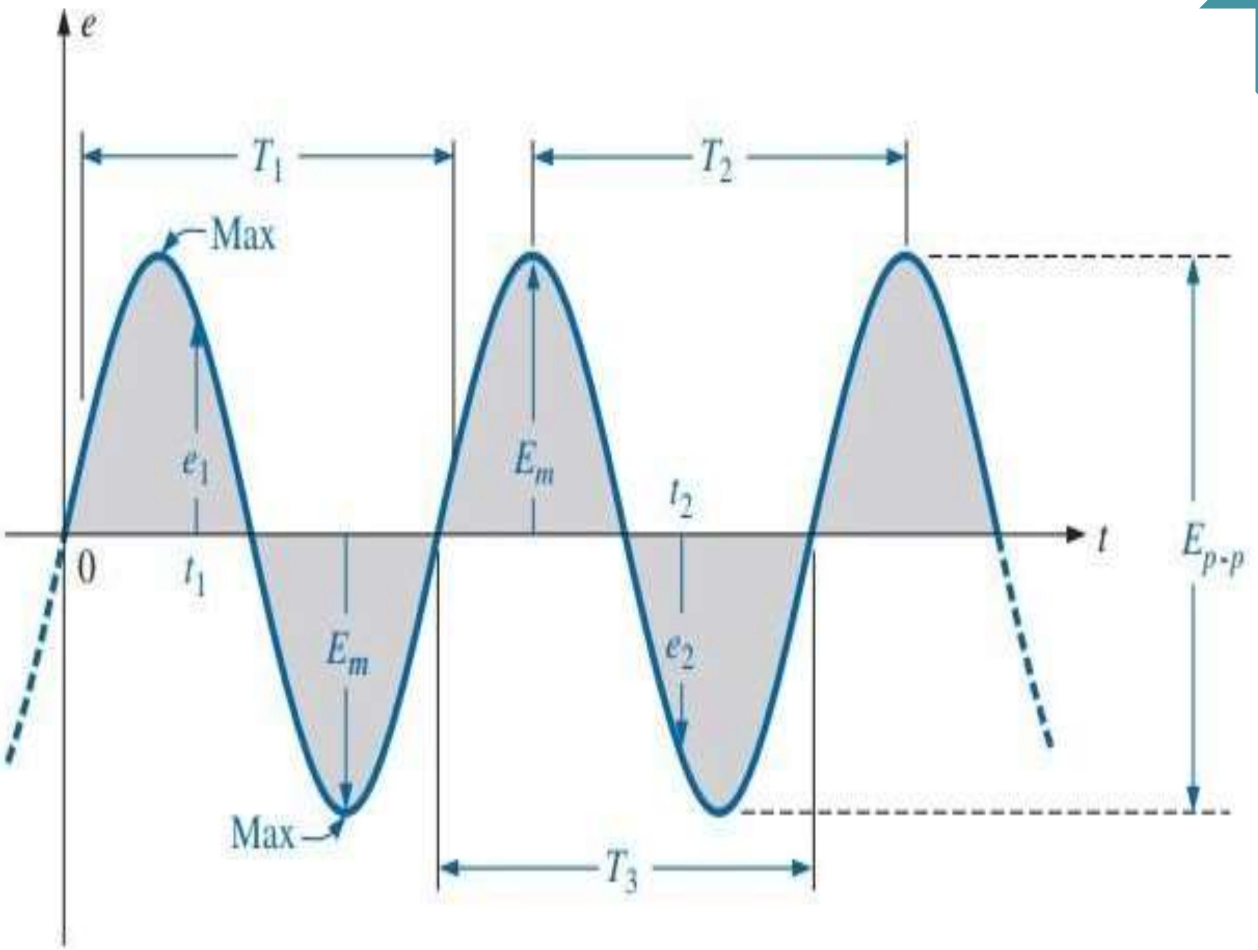
Instantaneous, average, and rms value in sinusoidal excitation



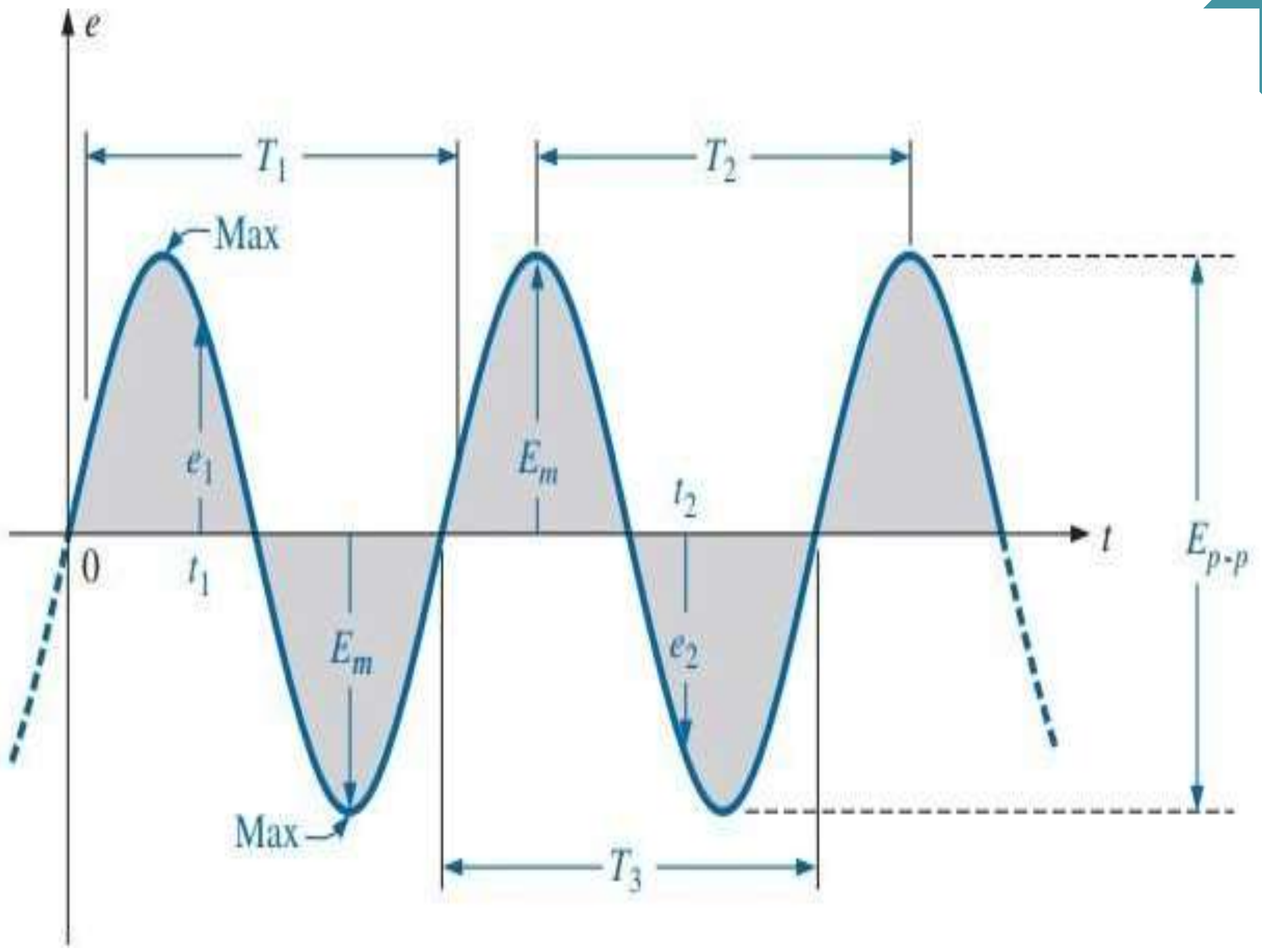
Definitions

Waveform: The path traced by a quantity, such as the voltage in Fig., plotted as a function of some variable such as time (as above), position, degrees, radians, temperature, and so on.



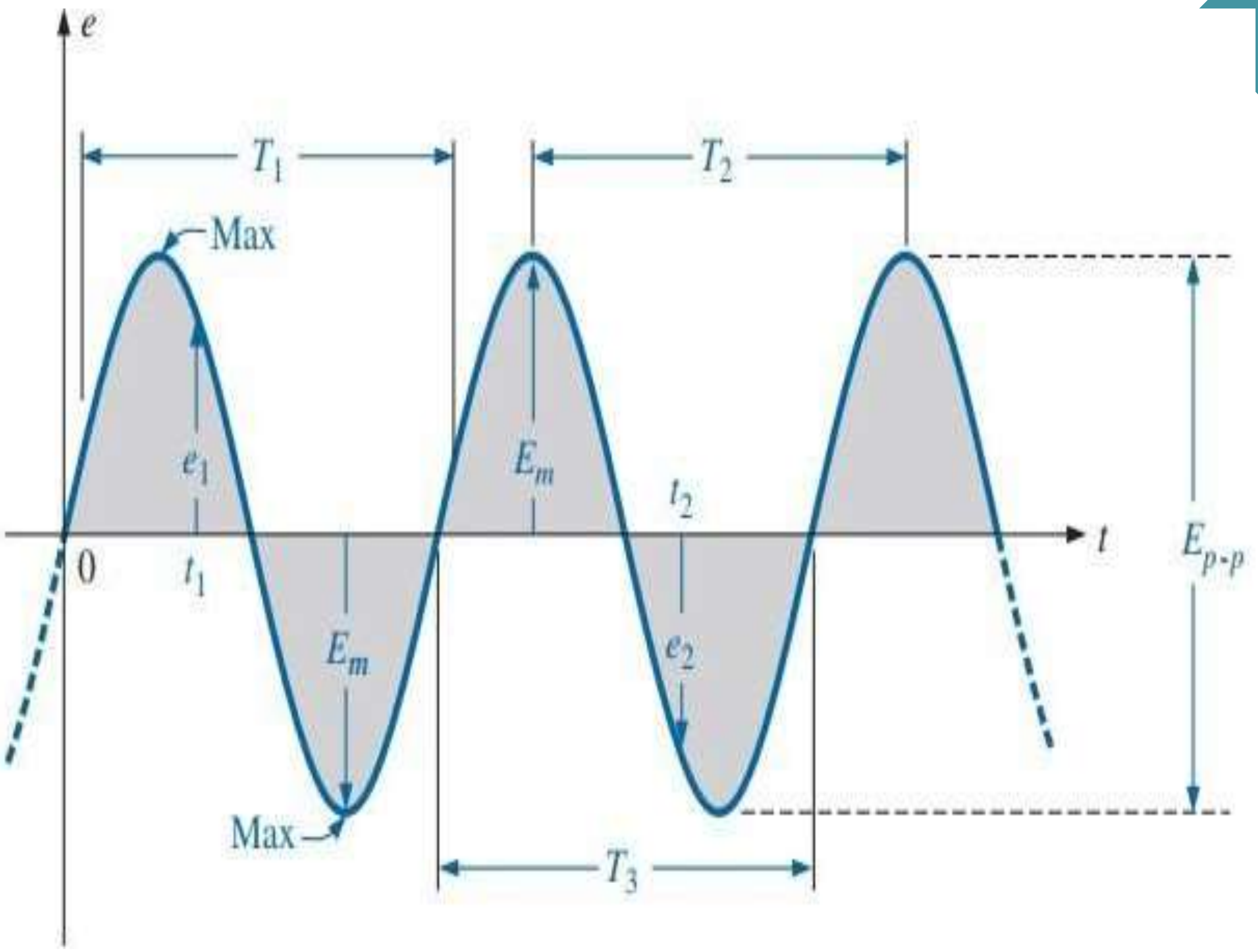
Definitions

Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e1, e2 in Fig.)



Definitions

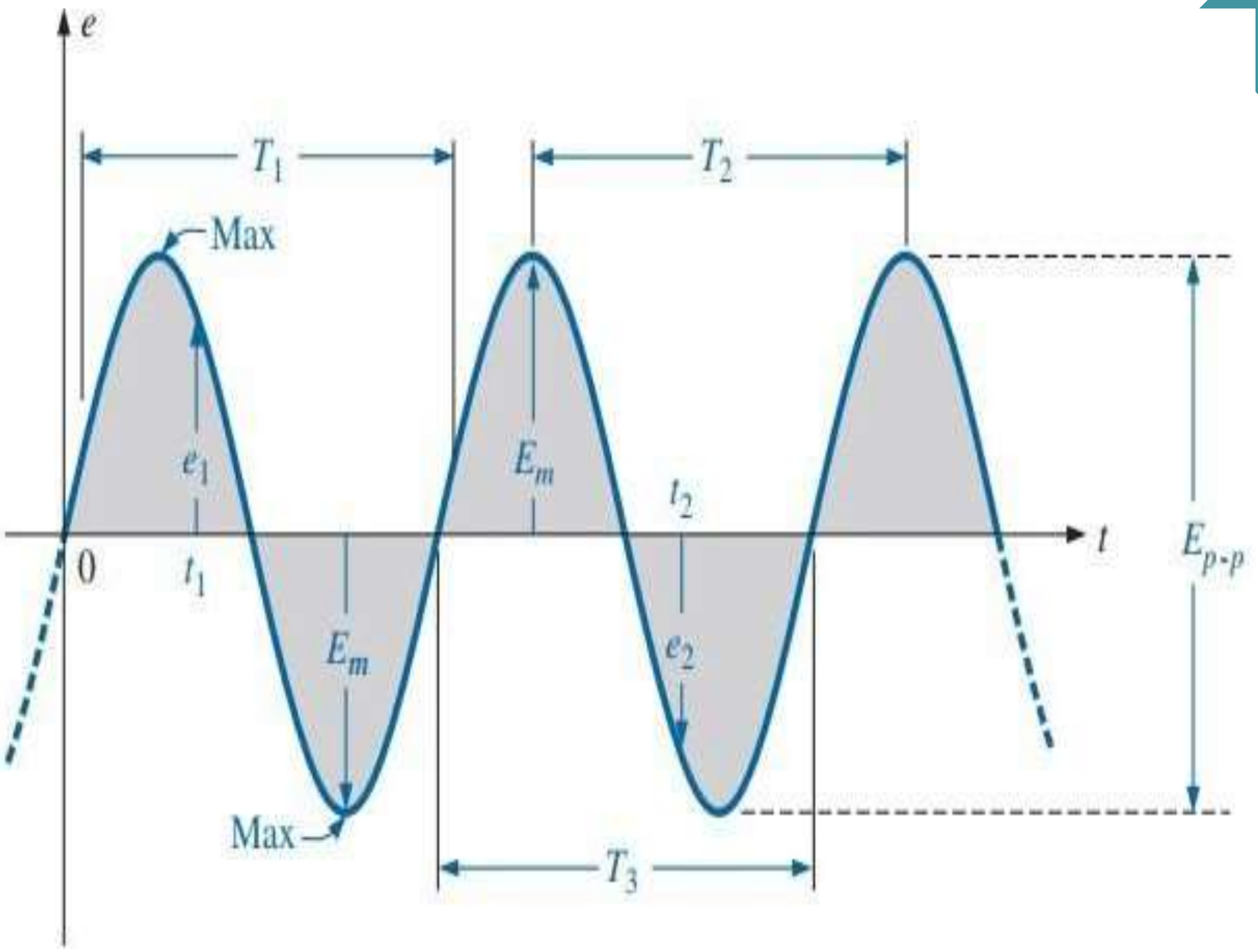
Peak amplitude: The maximum value of a waveform as measured from its average, or mean, value, denoted by uppercase letters such as Em (Fig.) for sources of voltage.



Definitions

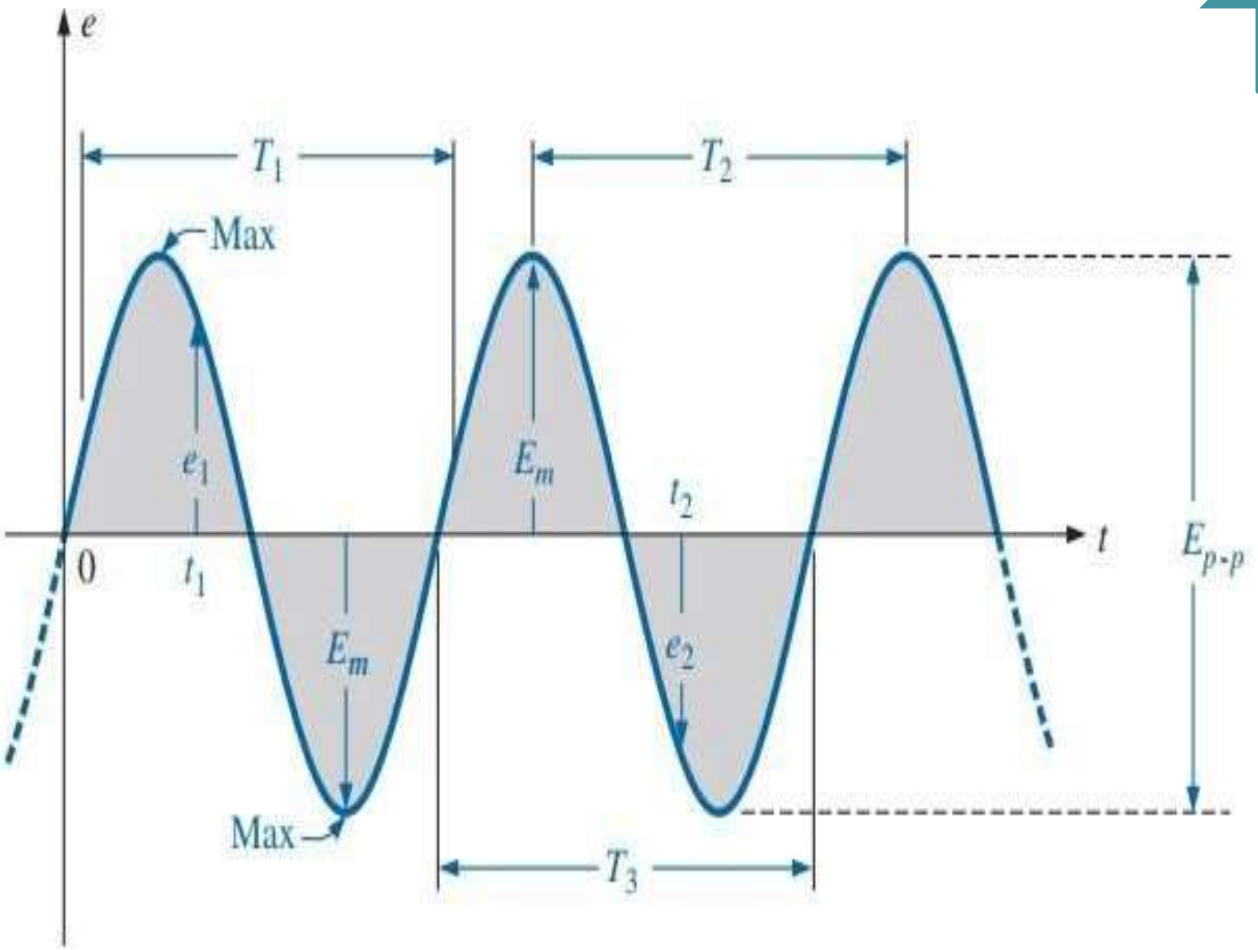
Peak-to-peak value:

Denoted by *Ep-p* or *Vp-p* , the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.



Definitions

Periodic waveform: A waveform that continually repeats itself after the same time interval

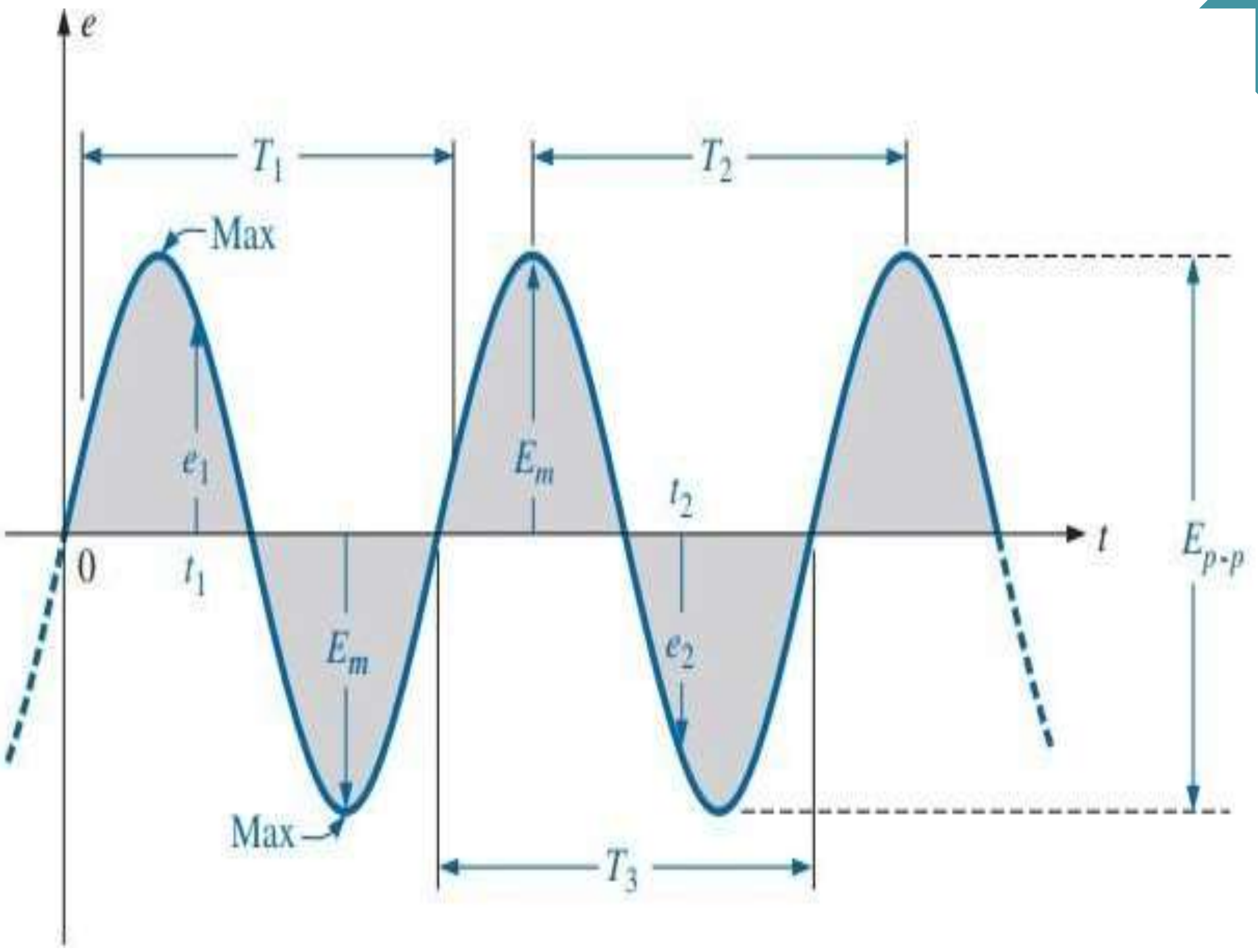


Definitions

Period (T): The time of a periodic waveform.

Cycle: The portion of a waveform contained in one period of time.

Frequency (f): The number of cycles that occur in 1 s .



Basics of Sinusoidal

The velocity with which the radius vector rotates about the center, called the **angular velocity,** can be determined from the following equation:

$$\boxed{\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}}}$$

$$\boxed{\omega = \frac{\alpha}{t}}$$

and

$$\boxed{\alpha = \omega t}$$

$$\boxed{\omega = \frac{2\pi}{T}} \quad \text{(rad/s)}$$

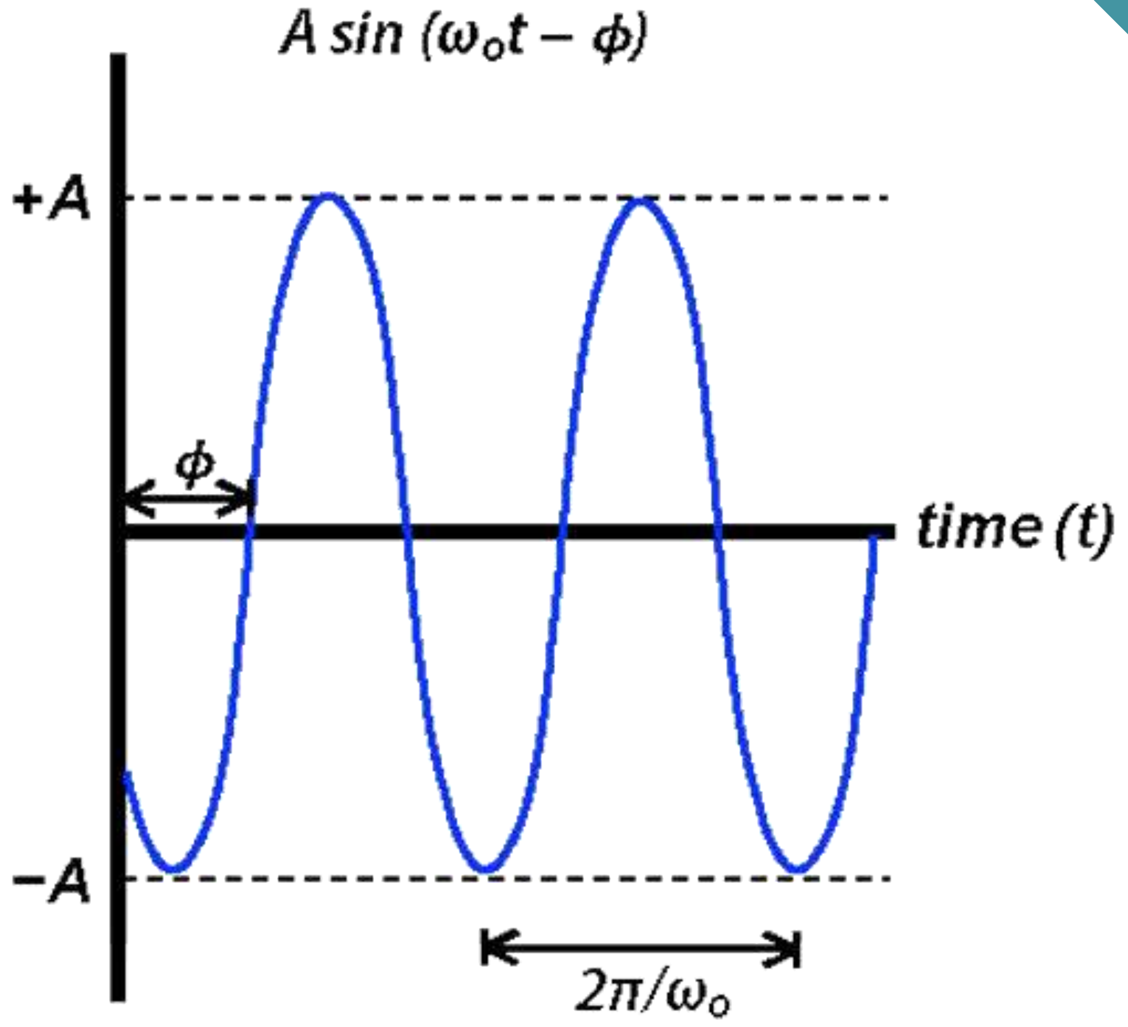
$$\boxed{\omega = 2\pi f} \quad \text{(rad/s)}$$

Question: Determine the angular velocity of a sine wave having a frequency of 60hz

Mathematical Expression of Sinusoidal

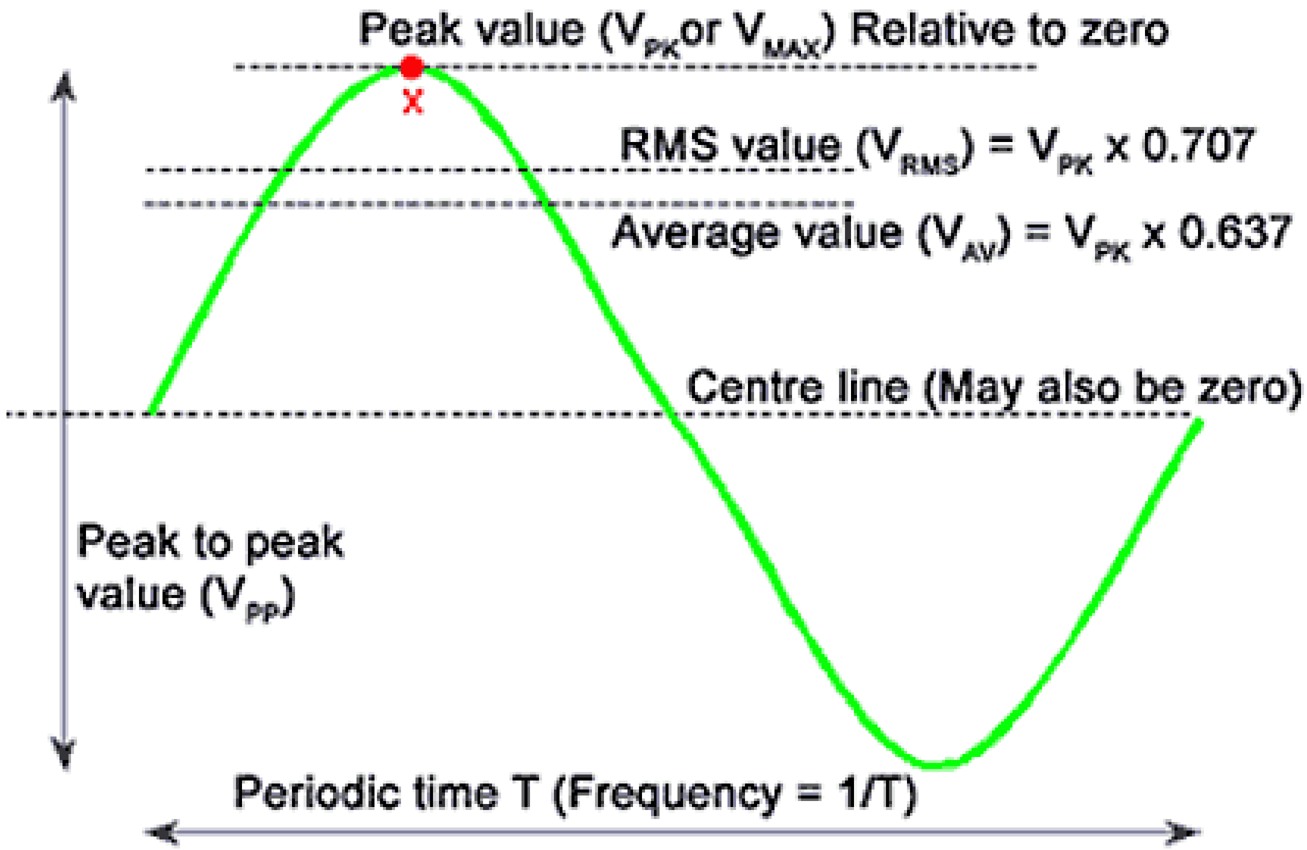
$$i = I_m \sin \omega t = I_m \sin \alpha$$

$$e = E_m \sin \omega t = E_m \sin \alpha$$



Average value

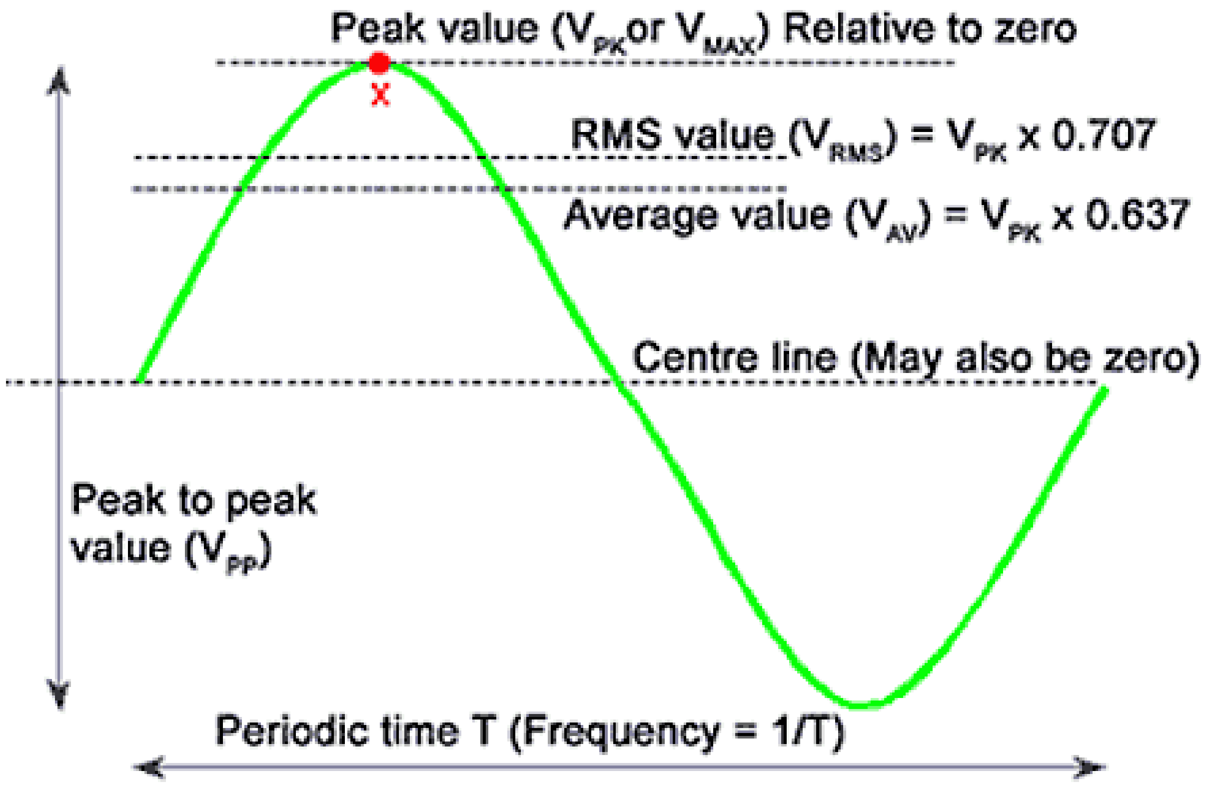
The average value of a sine wave is also known as the mean value. It's useful for calculating the equivalent DC value of rectified AC outputs.



Average value

The average value of a sine wave is calculated by: Integrating the area under the curve of one cycle and then dividing by the cycle's period.

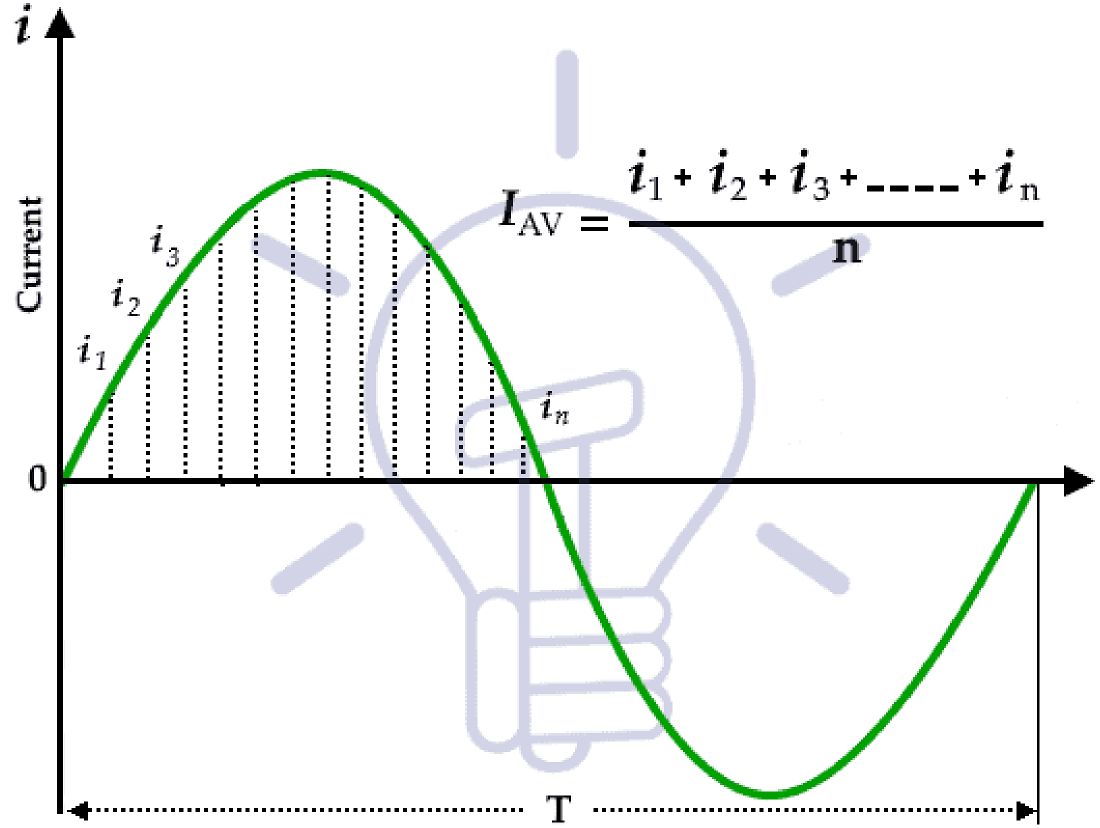
The average value of a sine wave over a complete cycle is zero because the positive and negative halves of the wave cancel each other out. However, the average value of a sine wave over half a cycle is 0.637 times the peak voltage.



Average value (Equation)

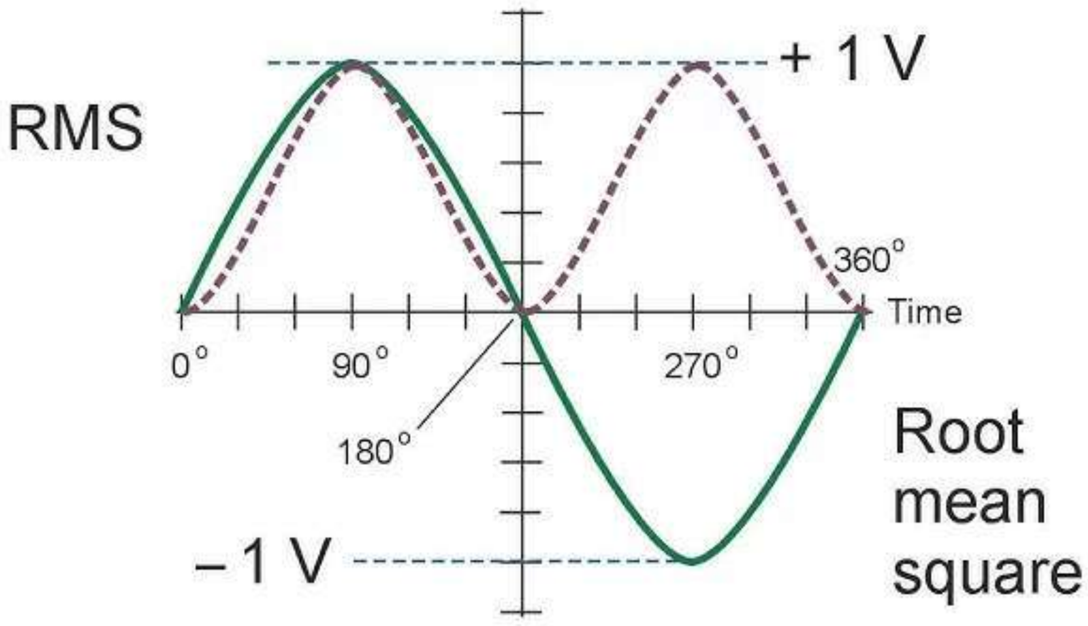
The average of all the instantaneous values of an alternating voltage and currents over one complete cycle is called.

$$I_{av} = \frac{i_1 + i_2 + i_3 + \dots\dots\dots + i_n}{n} = \frac{\text{Area of alternation}}{\text{Base}}$$



R.M.S value

- RMS stands for Root Mean Square
- This value represents the "effective" value of a sine wave, essentially the equivalent DC voltage that would produce the same amount of power in a resistor.



R.M.S value (Mathematical Equation)

- For a sine wave, the RMS value is always 0.707 times the peak value due to the mathematical properties of the sine function.

Mean value of $i_2 = \frac{1}{\pi}\int_0^{\pi} i^2\, d\theta$

$$(I_{RMS})^2 = \frac{1}{\pi}\int_0^{\pi} I_m^2 \, Sin^2\, \theta.d\theta = \frac{(I_m)^2}{\pi}\int_0^{\pi} Sin^2\, \theta.d\theta$$

$$= \frac{(I_m)^2}{\pi}\int_0^{\pi} \frac{1}{2}(1\text{-}Cos\ 2\theta)d\theta = \frac{(I_m)^2}{2\pi}\left[\theta - \frac{Sin\, 2\theta}{2}\right]_0^{\pi}$$

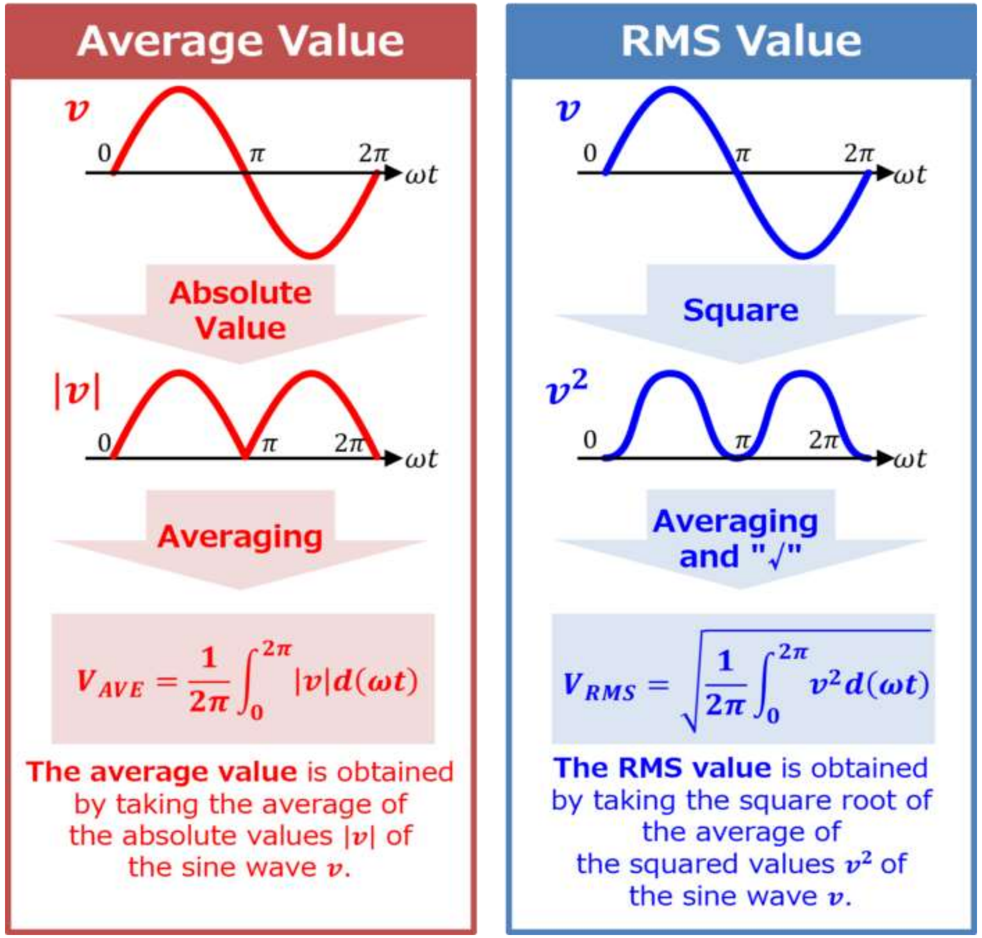
$$= \frac{(I_m)^2}{2\pi}\left[\left(\pi - \frac{Sin\ 2\pi}{2}\right) - \left(\frac{0 - Sin\ 2 \times 0}{2}\right)\right]$$

$$= \frac{(I_m)^2}{2\pi}\ [(\pi - 0) - (0 - 0)] = \frac{I_m^2}{2}$$

$$I_{RMS} = \frac{I_m}{\sqrt{2}} = \mathbf{0.707\ I_m}$$

$$\text{Similarly } E_{RMS} = \frac{E_m}{\sqrt{2}} = \mathbf{0.707\ E_m}$$

Average value Vs. R.M.S value



Different Factors

Waveform	RMS value	Crest factor	Form Factor
Sine wave	$\frac{1}{\sqrt{2}}$or 0.7071	$\sqrt{2}$or 1.414	1.11
Triangular Wave	$\frac{1}{\sqrt{3}}$ or 0.577	$\sqrt{3}$or 1.732	1.154
Square Wave	1	1	1
Saw tooth Wave	$\frac{1}{\sqrt{3}}$ or 0.577	$\sqrt{3}$ or 1.732	1.154

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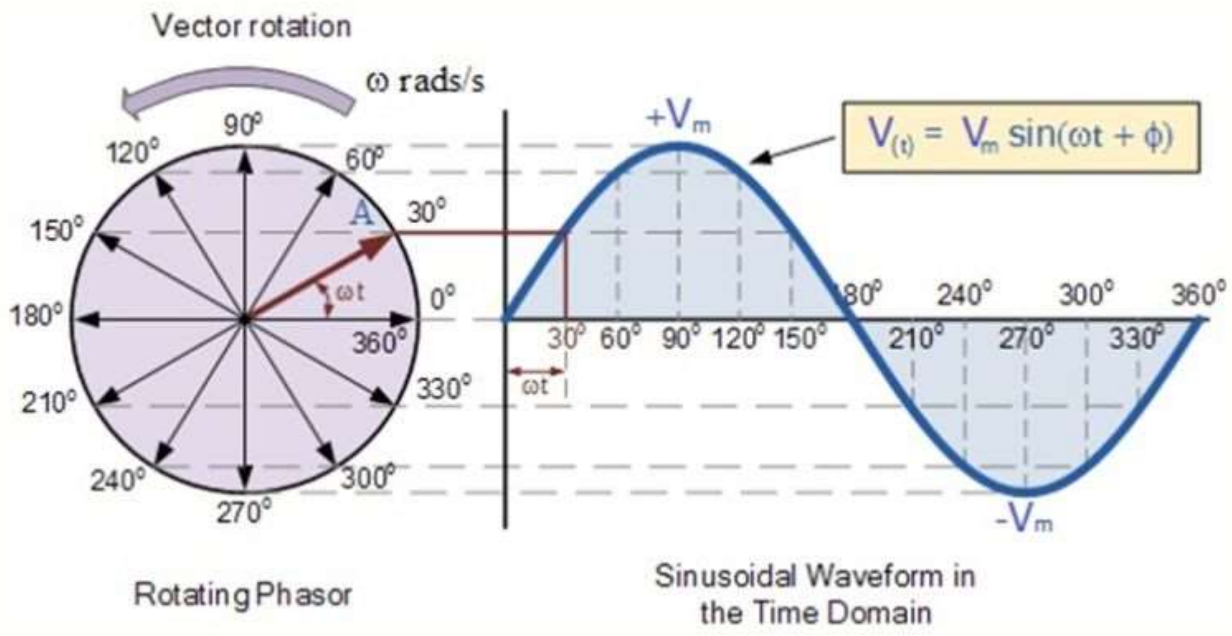
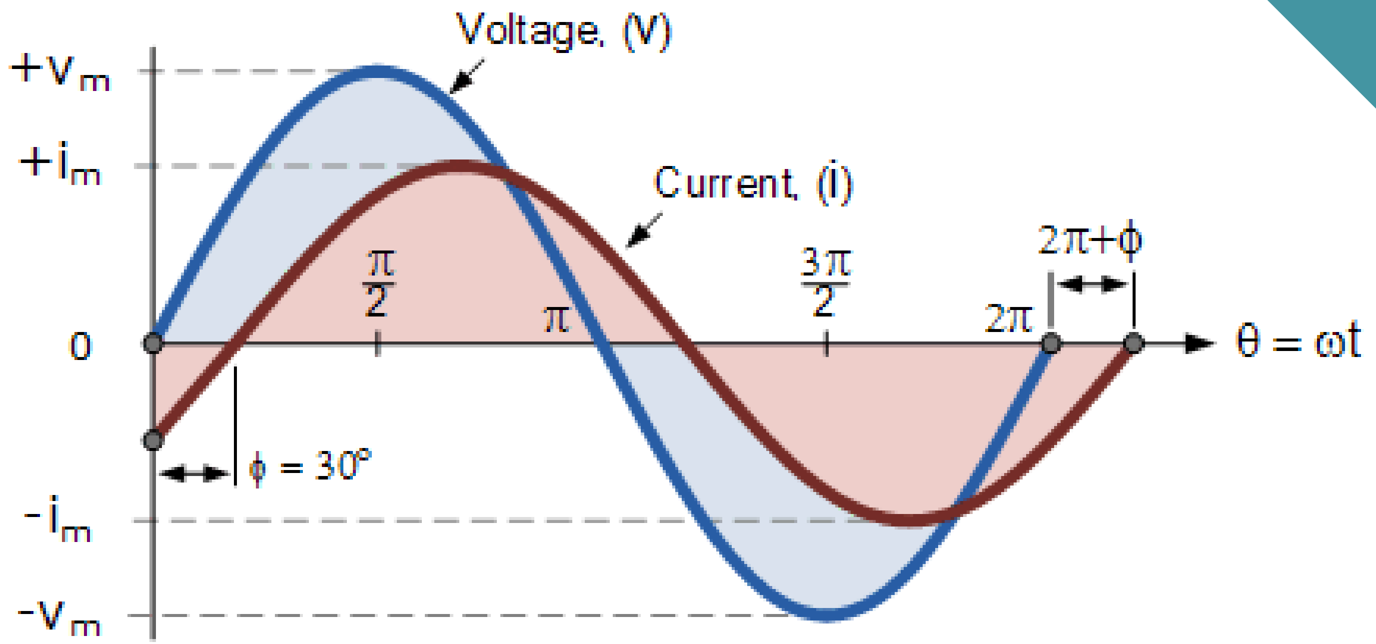
Complex exponential forcing functions and phasors

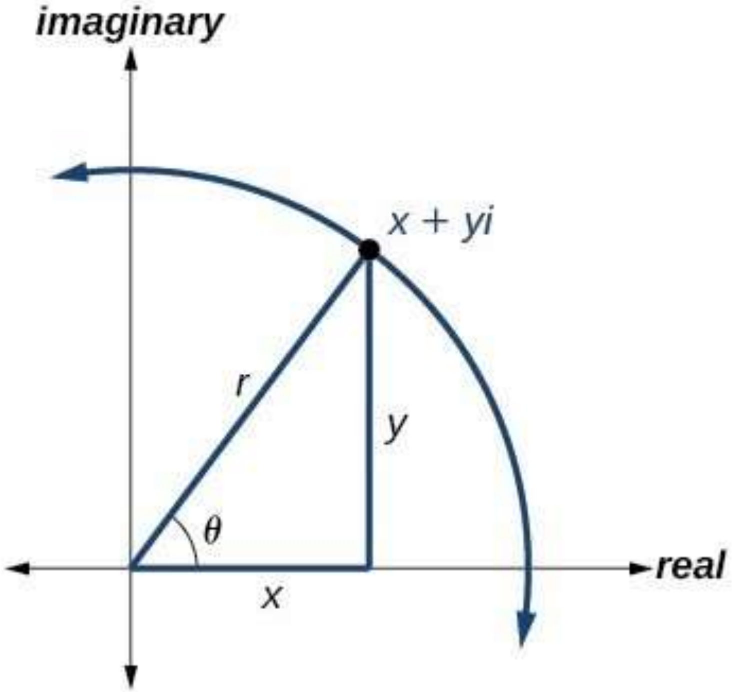


What is phasor?

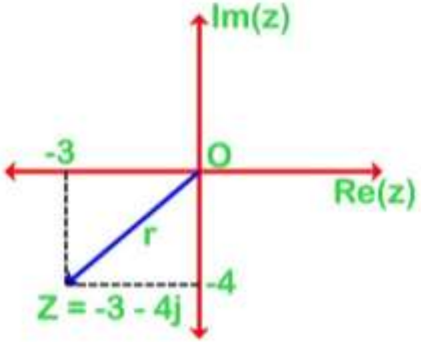
A **phasor** is a complex number that represents the amplitude and phase of a sinusoid.



Complex Number System



$$Z = -3 - 4j$$



$$Z = r\angle\theta$$

$$\theta = tan^{-1}\left|\frac{y}{x}\right|$$

$$r = \sqrt{x^2 + y^2}$$

Polar form a complex number

Complex Number Operation

$$\begin{array}{r} 2 + j5 \\ + \quad 4 - j3 \\ \hline \mathbf{6 + j2} \end{array} \qquad \begin{array}{r} 175 - j34 \\ + \quad 80 - j15 \\ \hline \mathbf{255 - j49} \end{array} \qquad \begin{array}{r} -36 + j10 \\ + \quad 20 + j82 \\ \hline \mathbf{-16 + j92} \end{array}$$

$$(35\angle 65^\circ)\,(10\angle -12^\circ) = \mathbf{350\angle 53^\circ}$$

$$(124\angle 250^\circ)\,(11\angle 100^\circ) = \mathbf{1364\angle -10^\circ}$$

or

$$\mathbf{1364\angle 350^\circ}$$

$$(3\angle 30^\circ)\,(5\angle -30^\circ) = \mathbf{15\angle 0^\circ}$$

Tips : Add or Subtract in Cartesian form But Multiplication or Division in Polar form

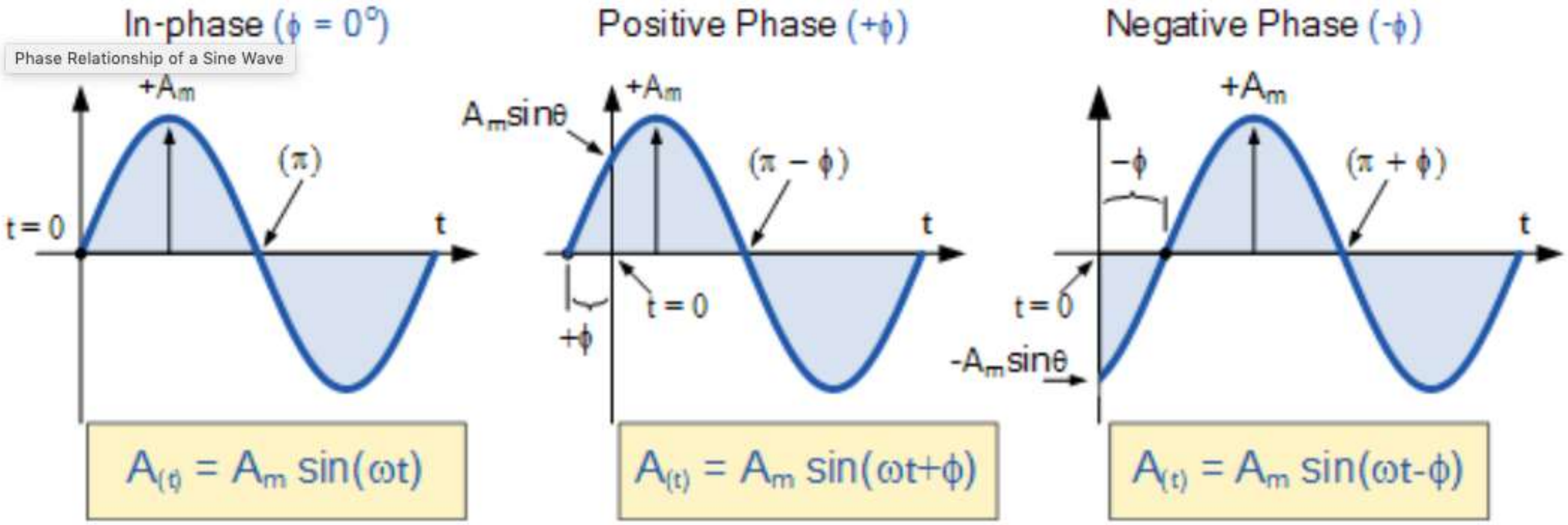
Representation of voltage & current in phasor

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad \mathbf{V} = V_m \angle \phi$$

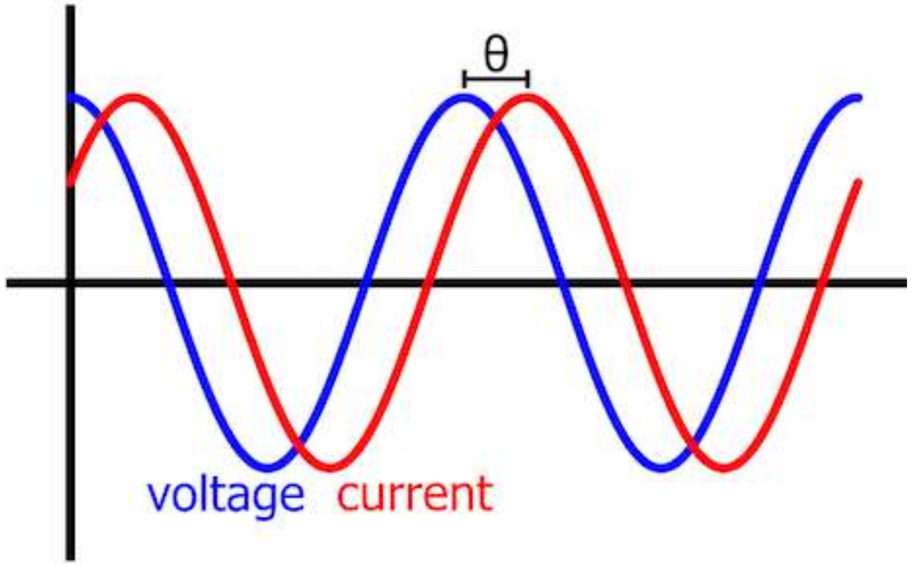
(Time-domain representation) (Phasor-domain representation)

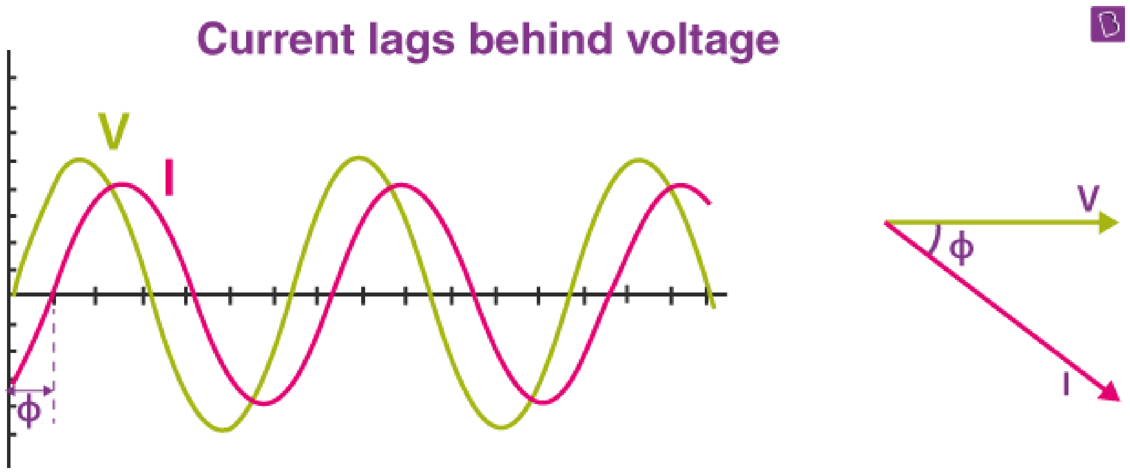
Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

Phase Relationship



Positive and Negative Phase (Lead and Lag)





Practice

Transform these sinusoids to phasors:

(a) $i = 6\cos(50t - 40^\circ)$ A
(b) $v = -4\sin(30t + 50^\circ)$ V

Solution:

(a) $i = 6\cos(50t - 40^\circ)$ has the phasor

$$\mathbf{I} = 6\angle -40^\circ \text{ A}$$

(b) Since $-\sin A = \cos(A + 90^\circ)$,

$$\begin{aligned} v = -4\sin(30t + 50^\circ) &= 4\cos(30t + 50^\circ + 90^\circ) \\ &= 4\cos(30t + 140^\circ) \text{ V} \end{aligned}$$

The phasor form of v is

$$\mathbf{V} = 4\angle 140^\circ \text{ V}$$

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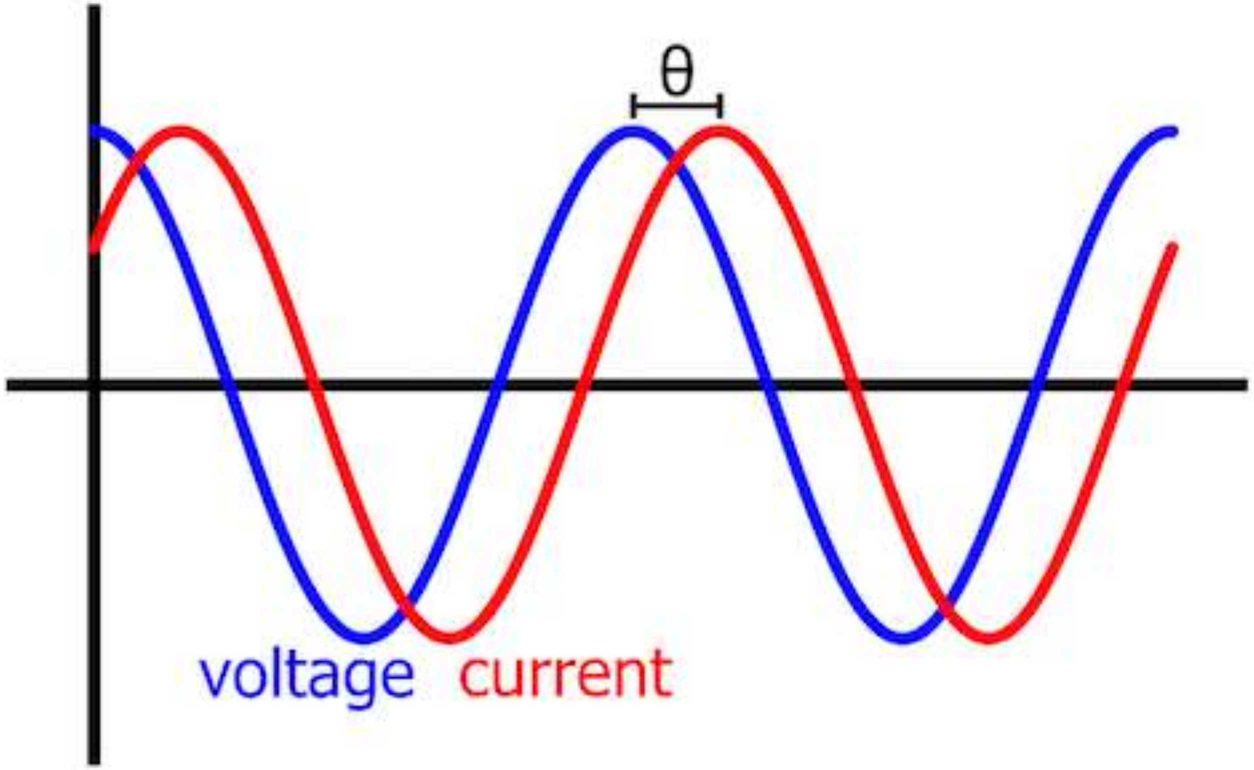
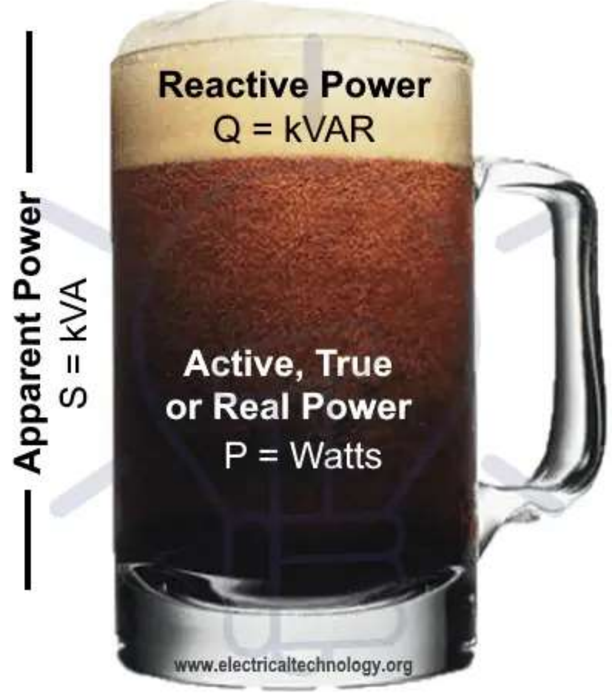
Different types of power and power factors for sinusoidal functions



Basics of apparent or complex power

- **Apparent power,** often represented as "S" in electrical engineering, is the total power seemingly drawn by a circuit.
- It is calculated by simply multiplying the RMS voltage and current.
- It includes both the "real power" (active power that does work) and "reactive power" (energy stored in inductors and capacitors that oscillates back and forth) within a circuit;



Beer Analogy of Active, Reactive & Apparent Power

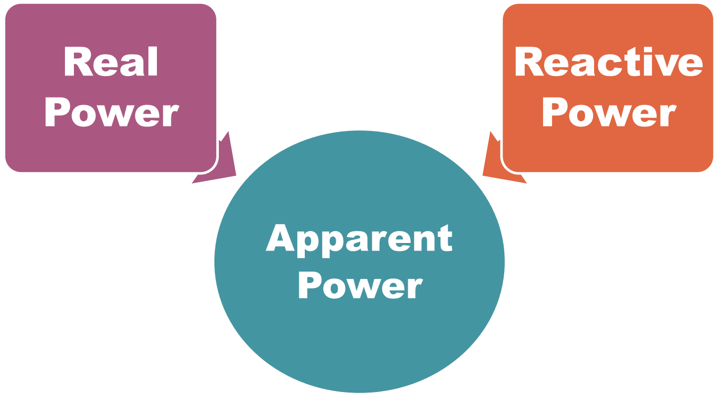
Basics of apparent or complex power

Key points about apparent power:

Calculation: Apparent power (S) is calculated by multiplying the RMS voltage (V) and RMS current (I): S = V * I.

Unit: Measured in volt-amperes (VA).

Relationship to real and reactive power: Apparent power is the hypotenuse of the power triangle, where the real power (P) is the adjacent side and reactive power (Q) is the opposite side.



Representation: Complex power is expressed as S = P + jQ, where "P" is the real power (active power) and "Q" is the reactive power.

Interpretation: The magnitude of complex power (|S|) is equal to the apparent power.

Power factor: The phase angle between voltage and current determines the power factor, which is calculated as cos(θ) where θ is the phase angle.

Basics of apparent or complex power

The complex power may be expressed in terms of the load impedance $\mathbf{Z}$. From Eq. (11.37), the load impedance $\mathbf{Z}$ may be written as

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{V_{\text{rms}}}{I_{\text{rms}}} \underline{/\theta_v - \theta_i} \tag{11.45}$$

Thus, $\mathbf{V}_{\text{rms}} = \mathbf{Z}\mathbf{I}_{\text{rms}}$. Substituting this into Eq. (11.41) gives

$$\boxed{\mathbf{S} = I_{\text{rms}}^2 \mathbf{Z} = \frac{V_{\text{rms}}^2}{\mathbf{Z}^*} = \mathbf{V}_{\text{rms}} \mathbf{I}_{\text{rms}}^*} \tag{11.46}$$

Since $\mathbf{Z} = R + jX$, Eq. (11.46) becomes

$$\mathbf{S} = I_{\text{rms}}^2 (R + jX) = P + jQ \tag{11.47}$$

where P and Q are the real and imaginary parts of the complex power; that is,

$$P = \text{Re}(\mathbf{S}) = I_{\text{rms}}^2 R \tag{11.48}$$

$$Q = \text{Im}(\mathbf{S}) = I_{\text{rms}}^2 X \tag{11.49}$$

Basics of Real power

Real power, also known as active power, is the power that an electrical system uses to perform work. It's measured in watts (W) and is the power that powers household appliances and machinery.

What it does

Real power is the power that powers devices like lightbulbs and phones. It's the power that's converted into heat, light, and motion.

Real Power

How it's calculated?

In DC circuits, real power is calculated using the formula P=I^2R) In AC circuits, the calculation also takes into account the phase difference between the voltage and current.

How it's related to other types of power?

Real power is one of three types of power: real power (P), reactive power (Q), and apparent power (S). The relationship between these three types of power can be represented using a power triangle

Basics of Real power

Real power is the power actually consumed due to the resistive load. The unit of real power is watt(w). It is denoted by P.

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$$

$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

$$P =$$

$$P = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$

Basics of Reactive power

Reactive power is the power that flows back to the grid in an alternating current (AC) system, and is also known as phantom power. It's different from active power, which is the power that's consumed by the load

How it works

In an AC system, reactive power flows back and forth between the phase conductors and the neutral conductor. This is because the system has different phases, which are caused by elements

Reactive Power

Types of reactive power

There are several types of reactive power, including inductive, capacitive, and harmonic.

Measurement

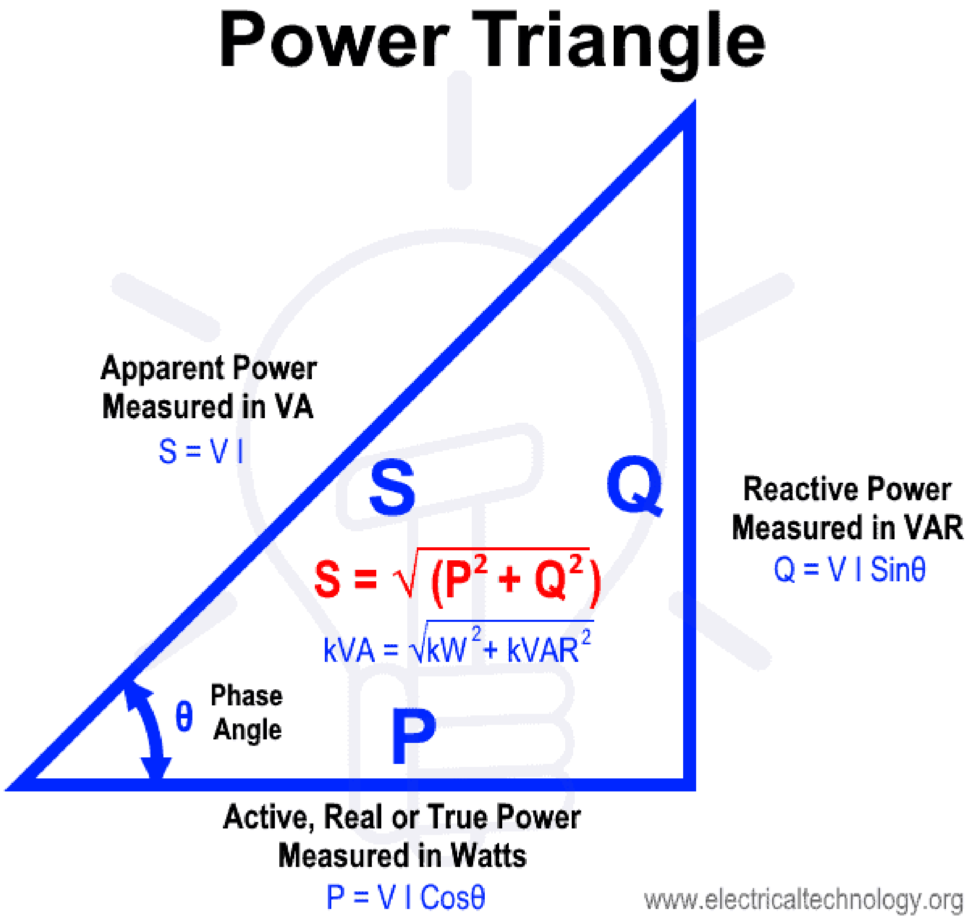
Reactive power is measured in volt-ampere-reactive (VAR).

Power factor

The ratio of real power to apparent power is called the power factor. A power factor less than one indicates reactive power is present in the system.

Power Triangle

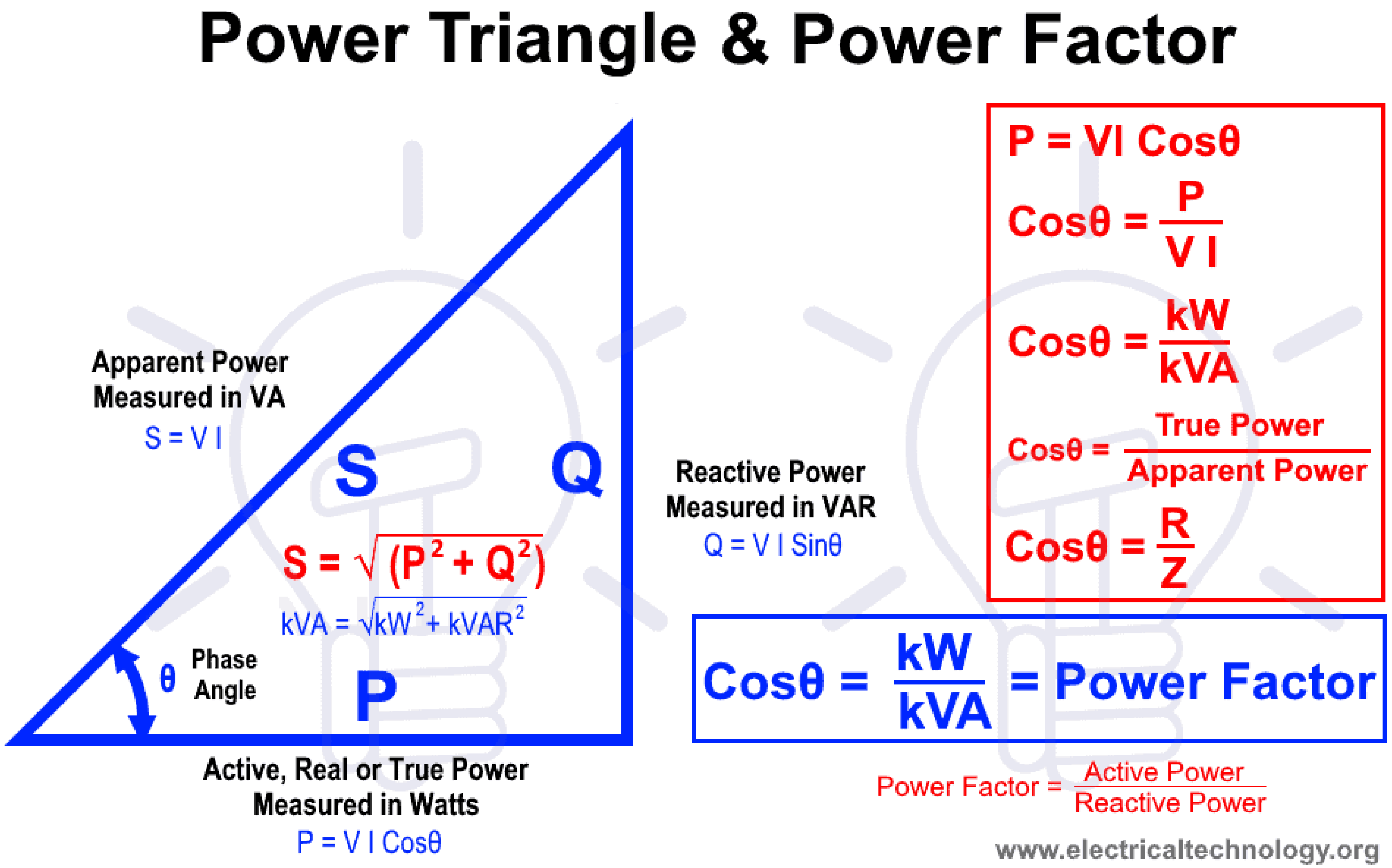
Power Triangle is a right angled triangle whose sides represent the active, reactive and apparent power. Base, Perpendicular and Hypogenous of this right angled triangle denotes the Active, Reactive and Apparent power respectively.



What is power factor?

Power factor is defined as the cosine of angle between the voltage phasor and current phasor in an AC circuit. It is denoted as pf. For an AC circuit, 0≤pf≤1 whereas for DC circuit power factor is always Unity(1).

Power factor (PF) is the ratio of working power to apparent power in an electrical system. It's a measure of how efficiently electrical power is converted into useful work



What is power factor?

Also power factor(pf) can be define as following,

1. Power factor, pf = $\cos(\theta v - \theta i)$

2. Cosθ = Active Power / Apparent Power

3. Cosθ = P / V I
4. Cosθ = P / S
5. Cosθ = kW / kVA
6. Cosθ = Resistance(R)/Impedance(Z)

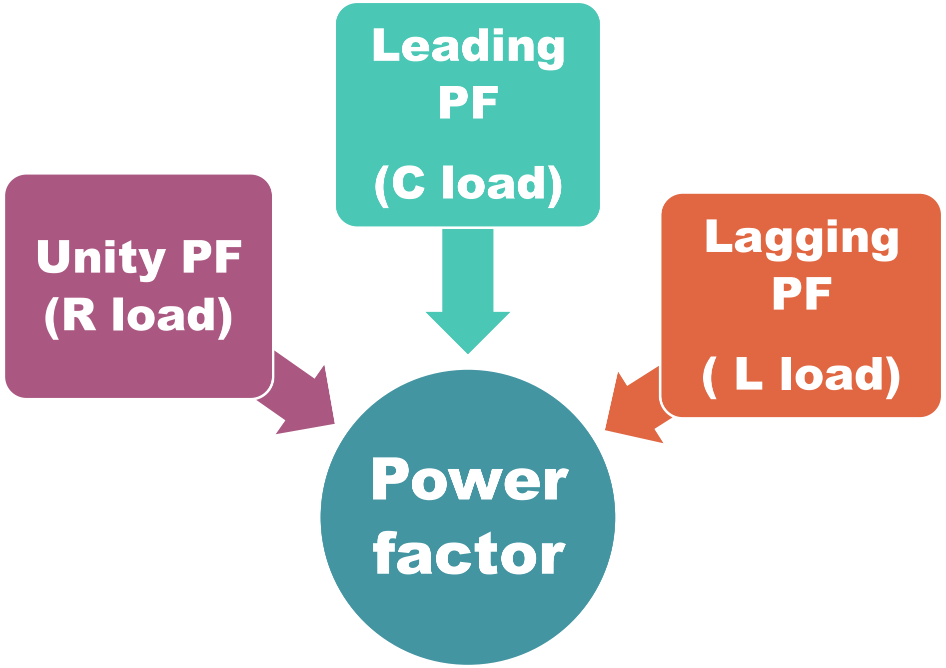
Types of Power factor

Types of power factor

Ideal power factor: The ideal power factor is 1, or unity.

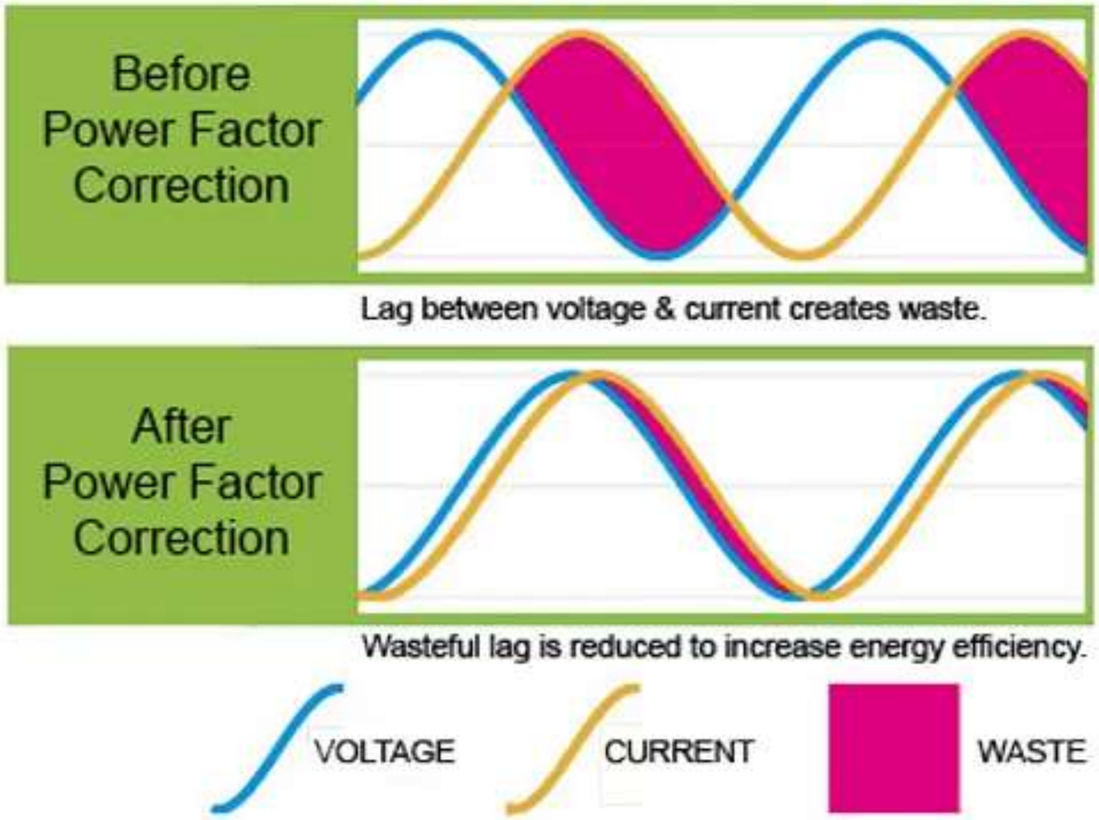
Leading power factor: This occurs when the current leads the voltage. Capacitive circuits, such as those with capacitor banks or synchronous condensers, have a leading power factor.

Lagging power factor: This occurs when the current lags behind the voltage.



Importance of Power factor

Power factor is important because it directly reflects the efficiency of an electrical system, indicating how effectively the supplied power is being utilized for useful work; a low power factor means more current is needed to deliver the same amount of power, leading to increased energy losses, higher electricity bills, and potential equipment damage, while a high power factor signifies optimal power usage and reduced system strain.



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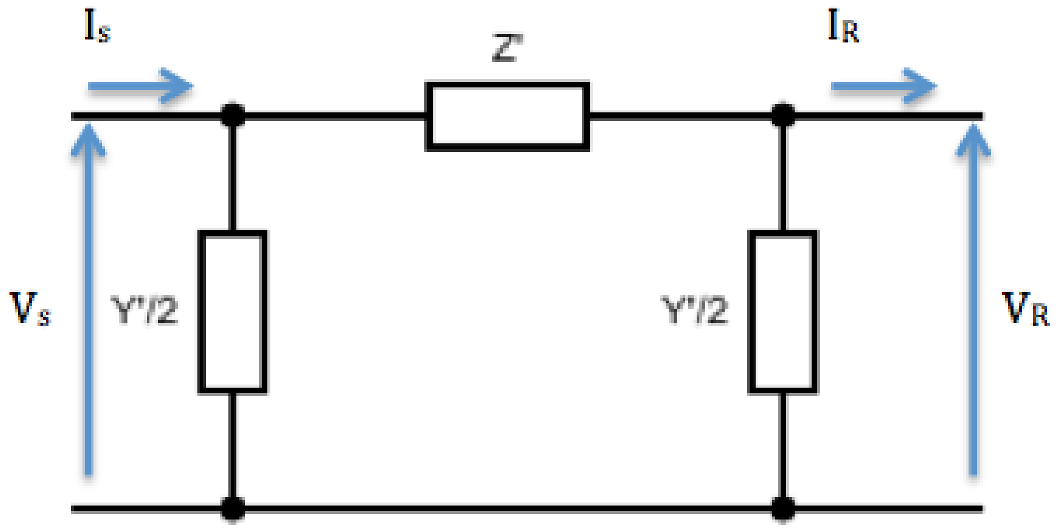
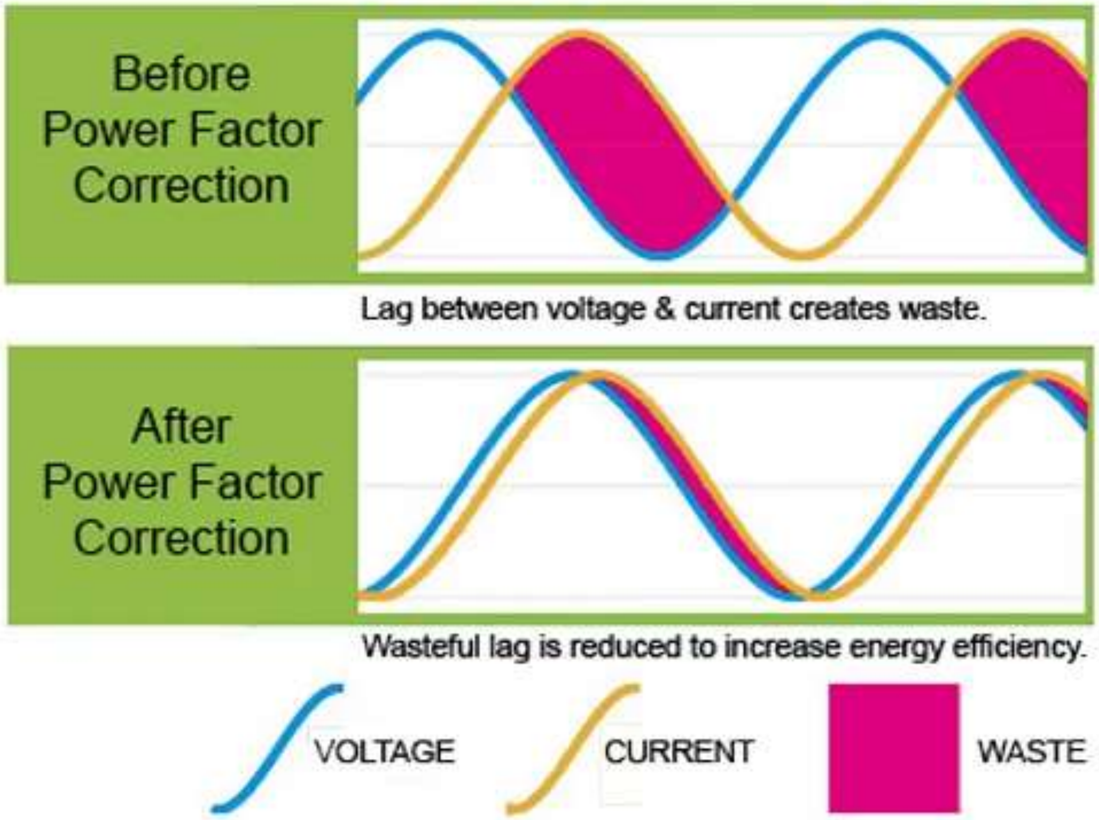
Impedance and admittance in AC circuits



Sinusoid & Phasor

Power factor is important because it directly reflects the efficiency of an electrical system, indicating how effectively the supplied power is being utilized for useful work; a low power factor means more current is needed to deliver the same amount of power, leading to increased energy losses, higher electricity bills, and potential equipment damage, while a high power factor signifies optimal power usage and reduced system strain.



Sinusoid & Phasor

A phasor is a complex number that represents the amplitude and phase of a sinusoid.

Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

Practice (Mathematics)

Find the amplitude, phase, period, and frequency of the sinusoid

$$v(t) = 12\cos(50t + 10^\circ)$$

Solution:

The amplitude is $V_m = 12$ V.
The phase is $\phi = 10^\circ$.
The angular frequency is $\omega = 50$ rad/s.
The period $T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1257$ s.
The frequency is $f = \frac{1}{T} = 7.958$ Hz.

Practice (Mathematics)

Calculate the phase angle between $v_1 = -10\cos(\omega t + 50°)$ and $v_2 = 12\sin(\omega t - 10°)$. State which sinusoid is leading.

Solution:
Let us calculate the phase in three ways. The first two methods use trigonometric identities, while the third method uses the graphical approach.

METHOD 1 In order to compare v_1 and v_2, we must express them in the same form. If we express them in cosine form with positive amplitudes,

$$v_1 = -10\cos(\omega t + 50°) = 10\cos(\omega t + 50° - 180°)$$

$$v_1 = 10\cos(\omega t - 130°) \quad \text{or} \quad v_1 = 10\cos(\omega t + 230°) \tag{9.2.1}$$

and

$$v_2 = 12\sin(\omega t - 10°) = 12\cos(\omega t - 10° - 90°)$$

$$v_2 = 12\cos(\omega t - 100°) \tag{9.2.2}$$

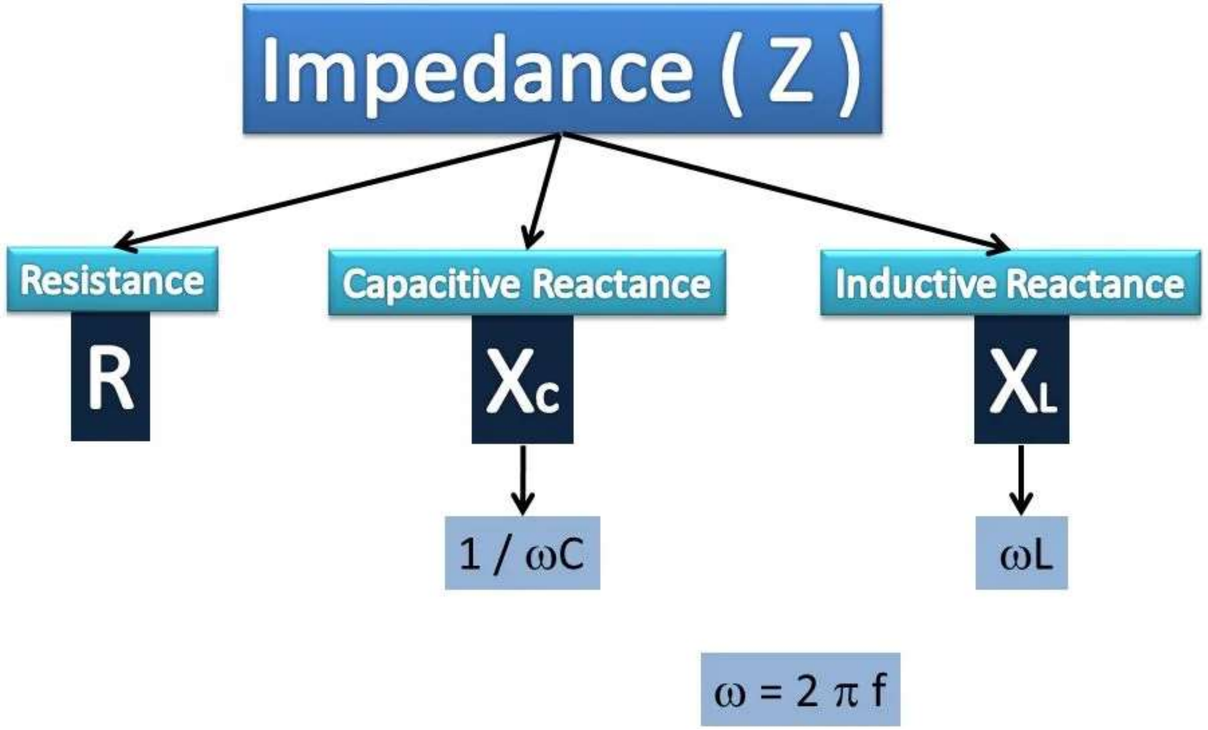
It can be deduced from Eqs. (9.2.1) and (9.2.2) that the phase difference between v_1 and v_2 is 30°. We can write v_2 as

$$v_2 = 12\cos(\omega t - 130° + 30°) \quad \text{or} \quad v_2 = 12\cos(\omega t + 260°) \tag{9.2.3}$$

Comparing Eqs. (9.2.1) and (9.2.3) shows clearly that v_2 leads v_1 by 30°.

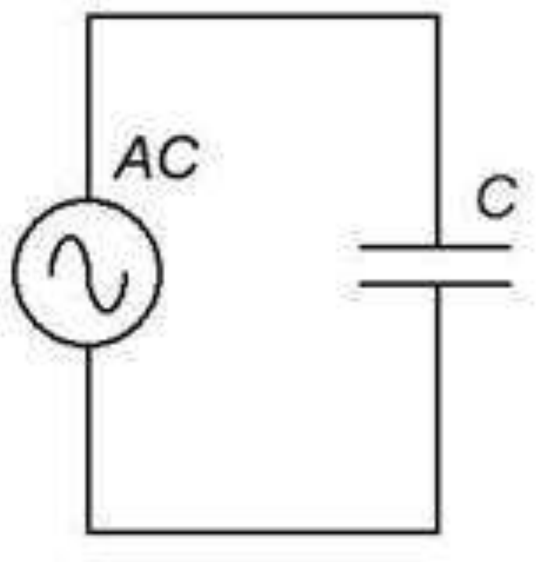
Impedance

In AC circuits, "impedance" measures how much a circuit resists the flow of alternating current (AC), combining both resistance and reactance, while "admittance" is the reciprocal of impedance, indicating how easily current can flow through a circuit, and is made up of conductance and susceptance;



Capacitive reactance

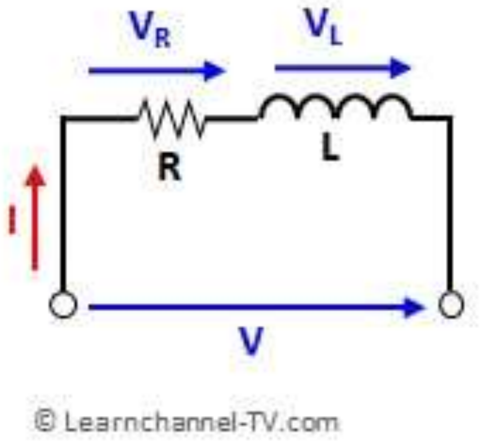
Capacitive reactance is the opposition offered by a capacitor to a change in current, while inductive reactance is the opposition offered by an inductor to a change in current; essentially, they both represent the resistance to alternating current flow within a circuit due to the specific properties of capacitors and inductors, with the key difference being that in a capacitor, current leads voltage, and in an inductor, current lags voltage.

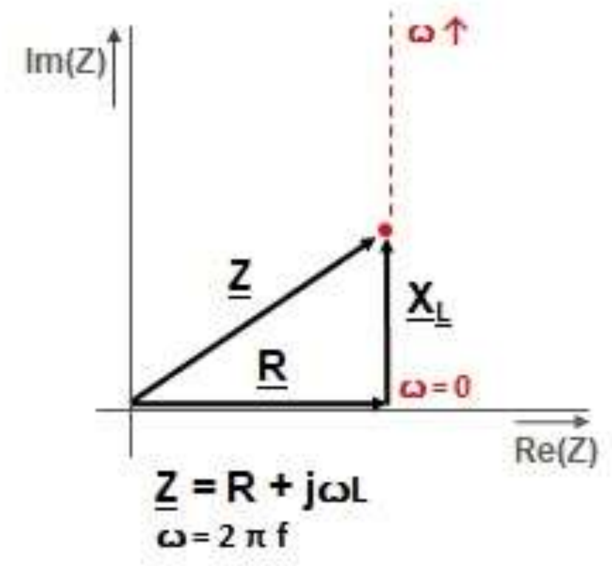


$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Inductive reactance

Inductive reactance is usually related to the magnetic field surrounding a wire or a coil carrying current. Likewise, capacitive reactance is often linked with the electric field that keeps changing between two conducting plates or surfaces that are kept apart from each other by some insulating medium.

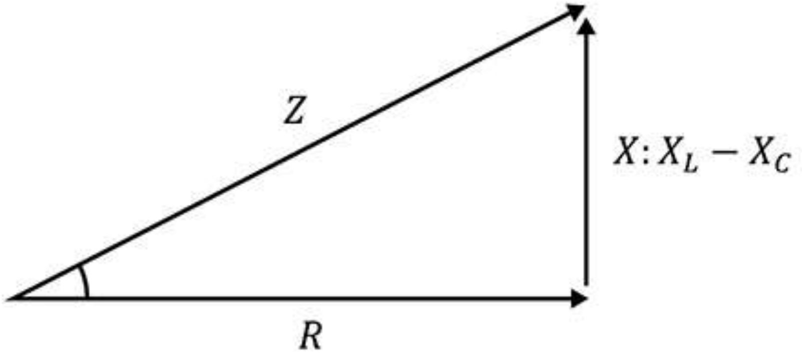




Impedance Triangle

An impedance triangle is a right triangle that represents the impedance of an AC circuit. The triangle's sides represent the resistance, reactance, and impedance of the circuit

Circuit element	Impedance	
	Symbol	Value
Resistor	R	R
Capacitor	X_C	$\frac{1}{j\omega C}$
Inductor	X_L	$j\omega L$



$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

Practice (Math)

Example 9.10

Find the input impedance of the circuit in Fig. 9.23. Assume that the circuit operates at $\omega = 50$ rad/s.

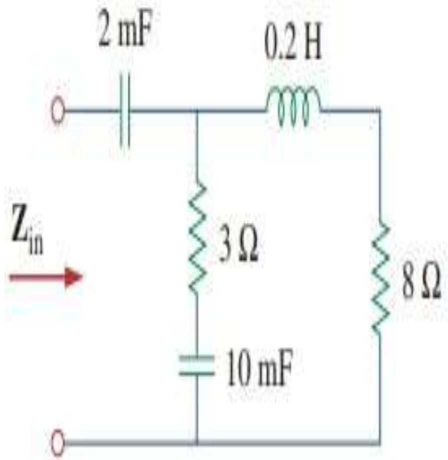


Figure 9.23
For Example 9.10.

Solution:

Let

$\mathbf{Z}_1$ = Impedance of the 2-mF capacitor

$\mathbf{Z}_2$ = Impedance of the 3-Ω resistor in series with the10-mF capacitor

$\mathbf{Z}_3$ = Impedance of the 0.2-H inductor in series with the 8-Ω resistor

Practice (Math)

Then

$$Z_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\,\Omega$$

$$Z_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\,\Omega$$

$$Z_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\,\Omega$$

The input impedance is

$$\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8}$$

$$= -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2} = -j10 + 3.22 - j1.07\,\Omega$$

Thus,

$$\mathbf{Z}_{in} = 3.22 - j11.07\,\Omega$$

Homework

Practice Problem 9.11 Calculate v_o in the circuit of Fig. 9.27.

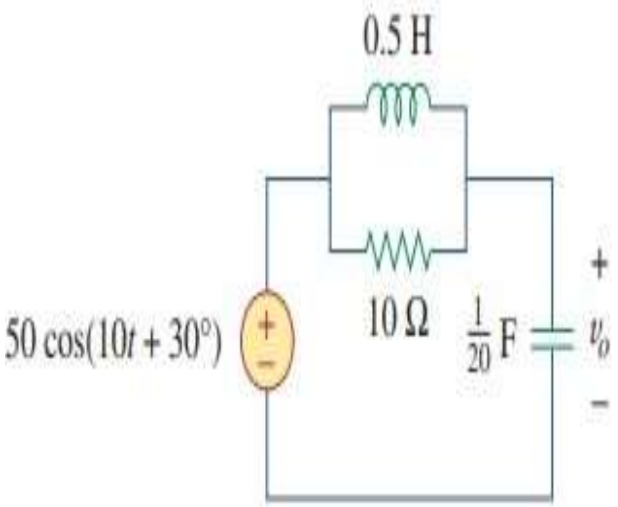


Figure 9.27

Answer: $v_o(t) = 35.36\cos(10t - 105°)$ V.

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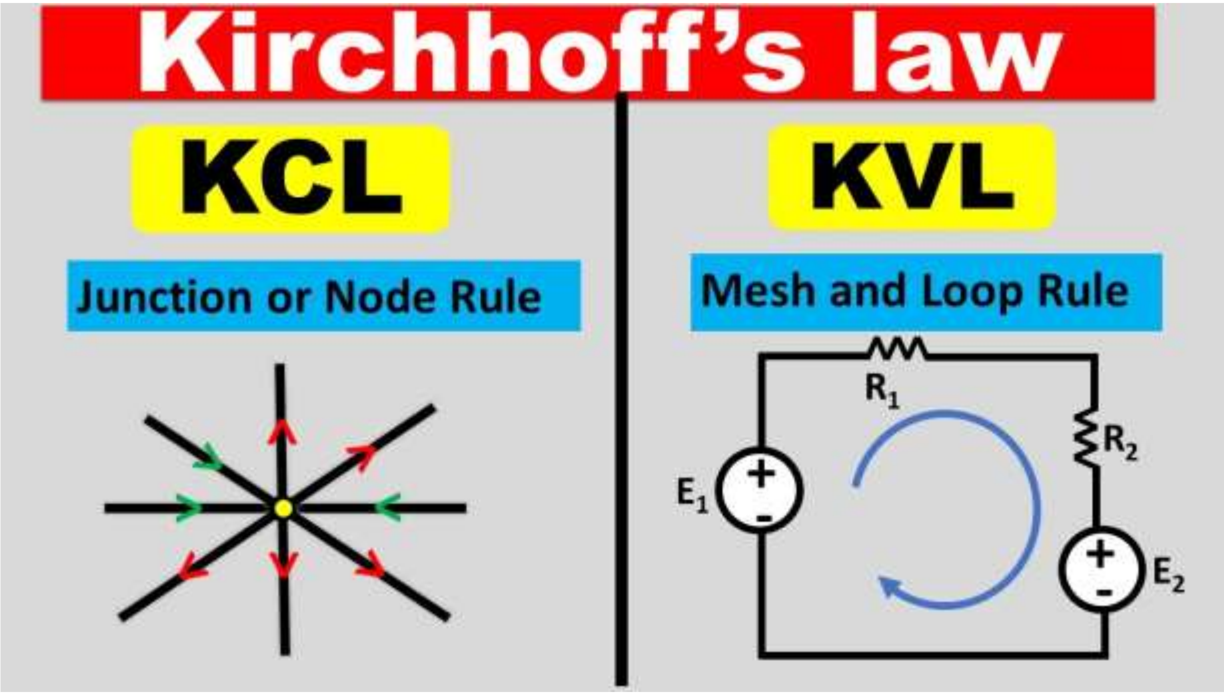
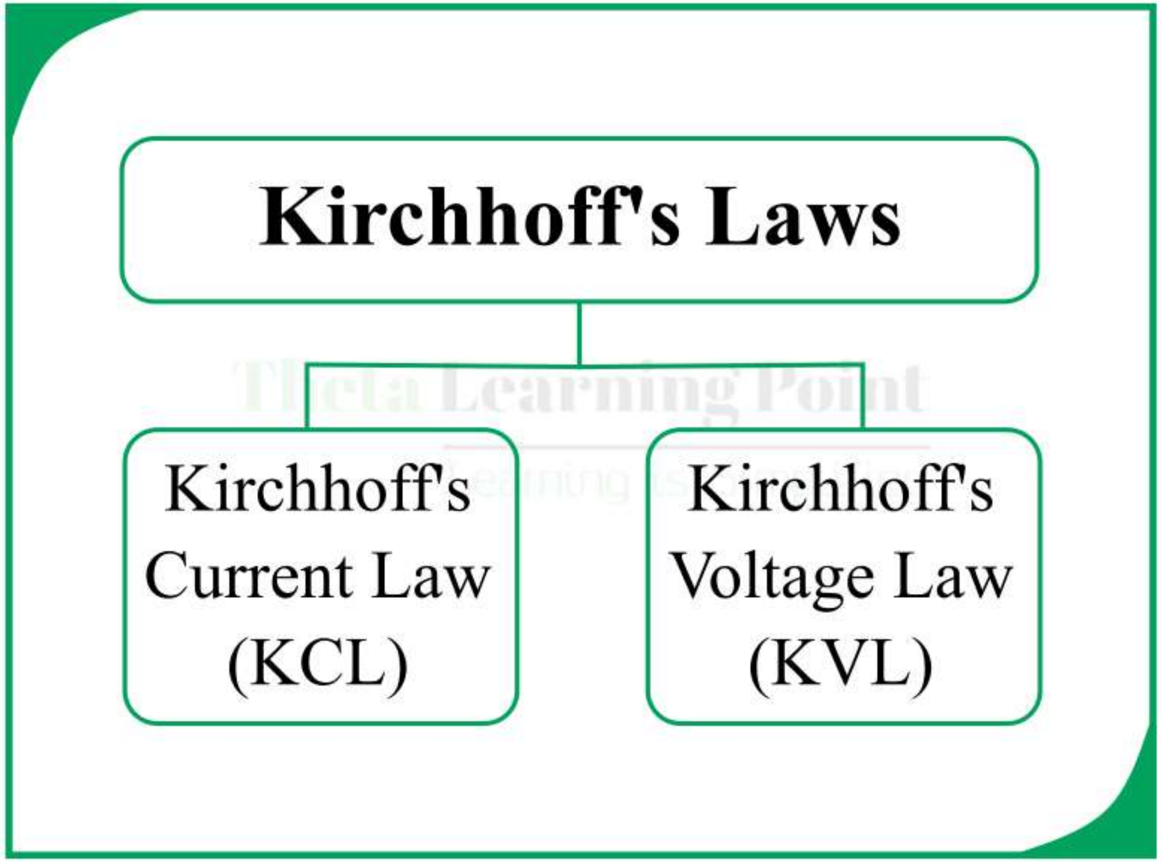
Basic circuit laws for AC circuits: KVL and KCL



KVL and KCL

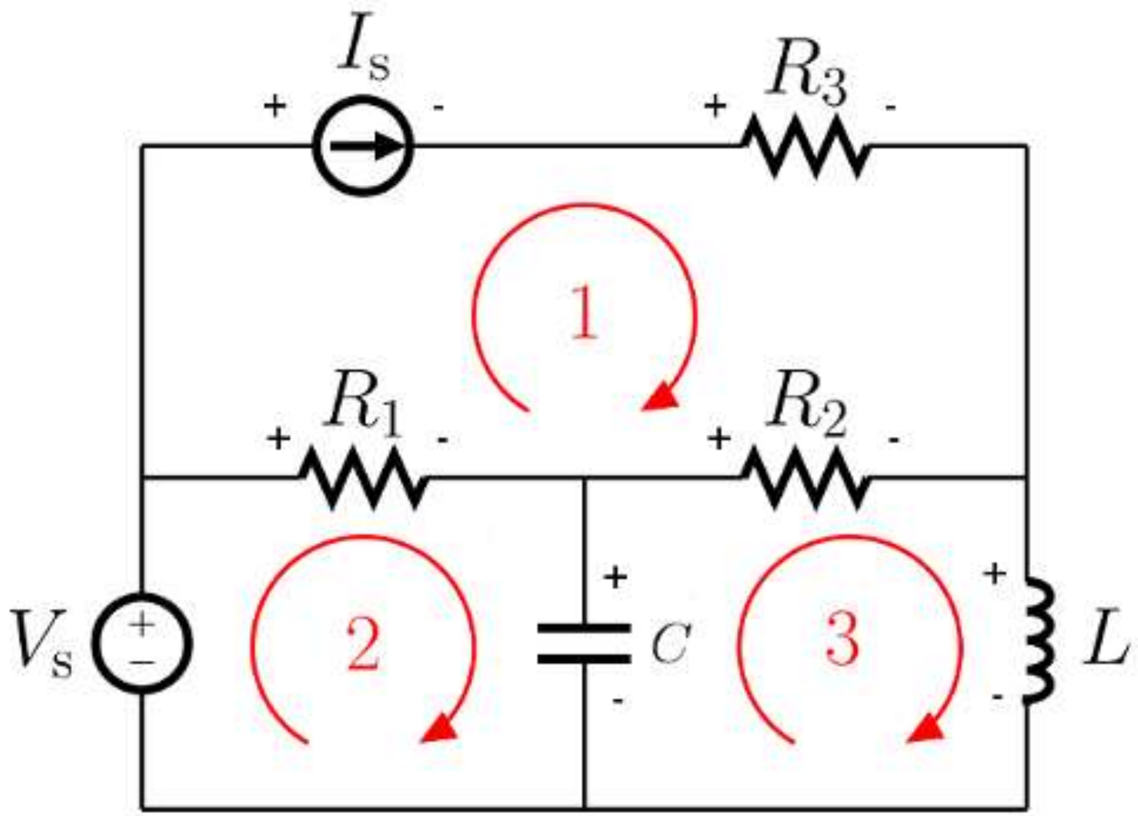


Kirchhoff's Laws
Kirchhoff's Current Law (KCL)
Kirchhoff's Voltage Law (KVL)

KVL

Kirchoff's Voltage Law (KVL) states that the sum of all voltage changes encountered along any closed path in an electric circuit is zero.

KVL is ultimately a statement of energy conservation; if it did not hold we could create electric circuits that produced unlimited amounts of energy for free.



KVL

Key steps in detail:

Identify a closed loop:

Select any complete path within the circuit where you can start and end at the same point.

Choose a loop direction:

Decide which way you will traverse the loop (clockwise or counterclockwise).

Assign voltage polarities:

For each component in the loop, determine the positive and negative voltage terminals based on the assumed current direction.

Write the KVL equation:

For each element in the loop, add the voltage drop (if going from positive to negative) or voltage rise (if going from negative to positive) with the appropriate sign.

Set the sum of all these voltage changes equal to zero.

KVL

Important points to remember:

Sign convention:

Consistent sign convention is crucial. If you move through a voltage source from the negative terminal to the positive terminal, consider it a positive voltage, and vice versa.

.

Current assumptions:

The voltage polarity assignments are based on the assumed current direction in the circuit.

Multiple loops:

If your circuit has multiple loops, apply KVL to each loop separately to generate a system of equations

KCL

To apply Kirchhoff's Current Law (KCL), follow these steps:

- **Identify the nodes:**

Locate all junctions or nodes in the circuit where multiple wires connect.

- **Assign current directions:**

Arbitrarily choose a direction for each current flowing through each branch connected to a node, indicating whether the current is entering or leaving the node.

- **Write the KCL equation:**

At each node, sum up all the currents entering the node and set it equal to the sum of all currents leaving the node, ensuring that the algebraic sum is zero.

- **Apply the sign convention:**

When writing the equation, use a positive sign for currents entering the node and a negative sign for currents leaving the node.

- **Solve the system of equations:**

If you have multiple nodes, you will have a system of equations that need to be solved simultaneously to find the unknown currents in the circuit.

KCL

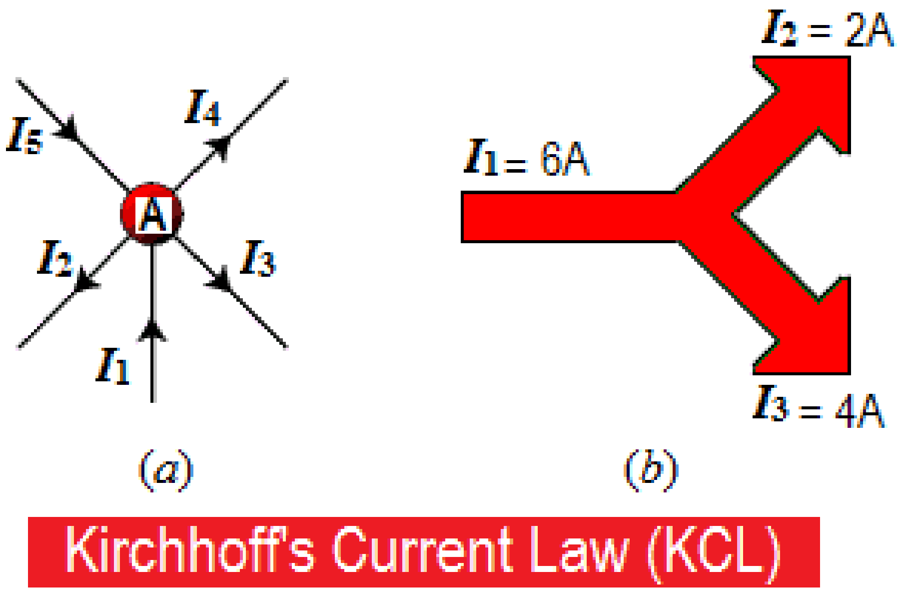
Key points about KCL:

•**Conservation of charge:**

KCL is based on the principle that charge cannot be created or destroyed at a junction, so the total current entering a node must equal the total current leaving it.

•**Mathematical representation:**

The equation for KCL can be written as: Σ(I_in) = Σ(I_out) where "I_in" represents currents entering the node and "I_out" represents currents leaving the node



Voltage & Current calculation (KVL, KCL)

Example 9.11 Determine $v_o(t)$ in the circuit of Fig. 9.25.

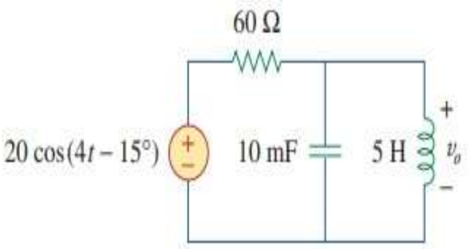


Figure 9.25
For Example 9.11.

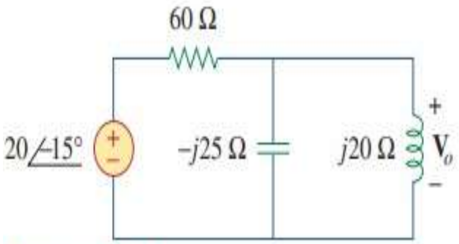


Figure 9.26
The frequency domain equivalent of the circuit in Fig. 9.25.

Solution:
To do the analysis in the frequency domain, we must first transform the time domain circuit in Fig. 9.25 to the phasor domain equivalent in Fig. 9.26. The transformation produces

$$v_s = 20\cos(4t - 15^\circ) \quad \Rightarrow \quad \mathbf{V}_s = 20\angle{-15^\circ}\ \text{V}, \qquad \omega = 4$$

$$10\ \text{mF} \quad \Rightarrow \quad \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}} = -j25\ \Omega$$

$$5\ \text{H} \quad \Rightarrow \quad j\omega L = j4 \times 5 = j20\ \Omega$$

Let

$\mathbf{Z}_1$ = Impedance of the 60-Ω resistor

$\mathbf{Z}_2$ = Impedance of the parallel combination of the 10-mF capacitor and the 5-H inductor

Then $\mathbf{Z}_1 = 60\ \Omega$ and

$$\mathbf{Z}_2 = -j25 \parallel j20 = \frac{-j25 \times j20}{-j25 + j20} = j100\ \Omega$$

By the voltage-division principle,

$$\mathbf{V}_o = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s = \frac{j100}{60 + j100}(20\angle{-15^\circ})$$

$$= (0.8575\angle{30.96^\circ})(20\angle{-15^\circ}) = 17.15\angle{15.96^\circ}\ \text{V}$$

We convert this to the time domain and obtain

$$v_o(t) = 17.15\cos(4t + 15.96^\circ)\ \text{V}$$

Homework:

Practice Problem 9.11 Calculate v_o in the circuit of Fig. 9.27.

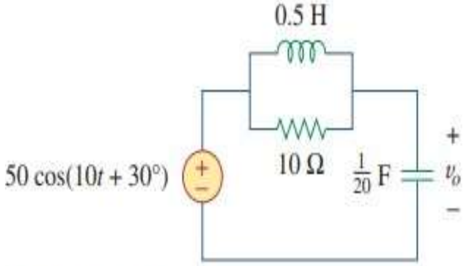


Figure 9.27

Answer: $v_o(t) = 35.36\cos(10t - 105^\circ)$ V.

WEEK-08
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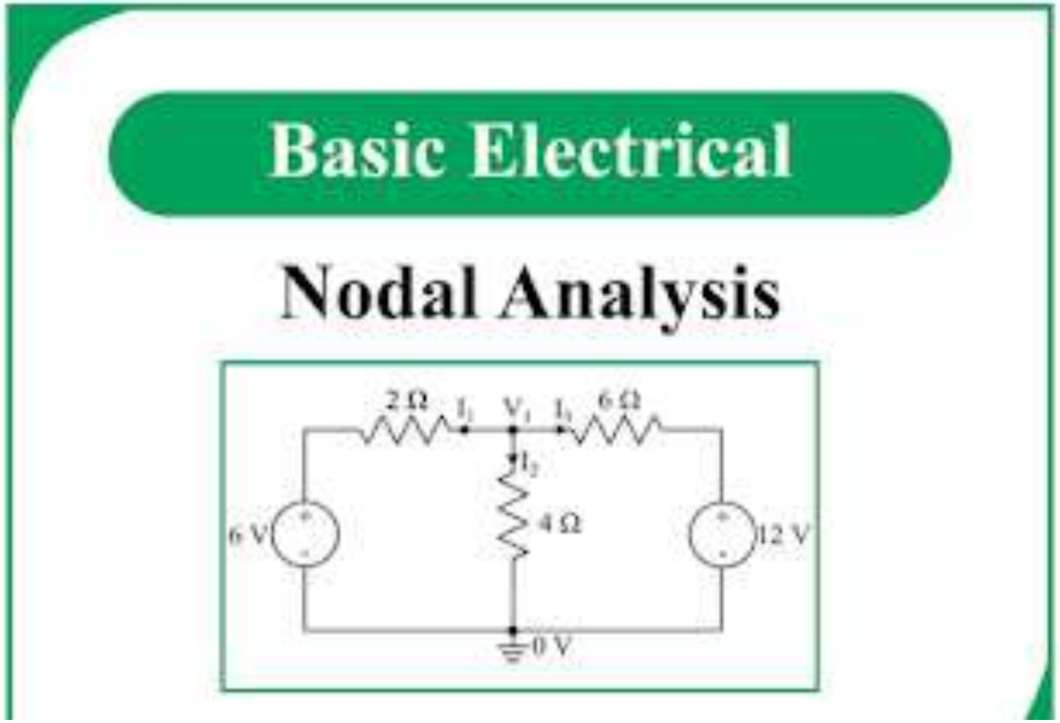
Nodal analysis in AC circuits



Nodal Analysis

The fundamental steps are the following:

•Determine the number of nodes within the network.

•Pick a reference node and label each remaining node with a subscripted value of voltage: V1V1, V2V2, and so on.

•Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law.

•Solve the resulting equations for the nodal voltages

Nodal Analysis

Find i_x in the circuit of Fig. 10.1 using nodal analysis.

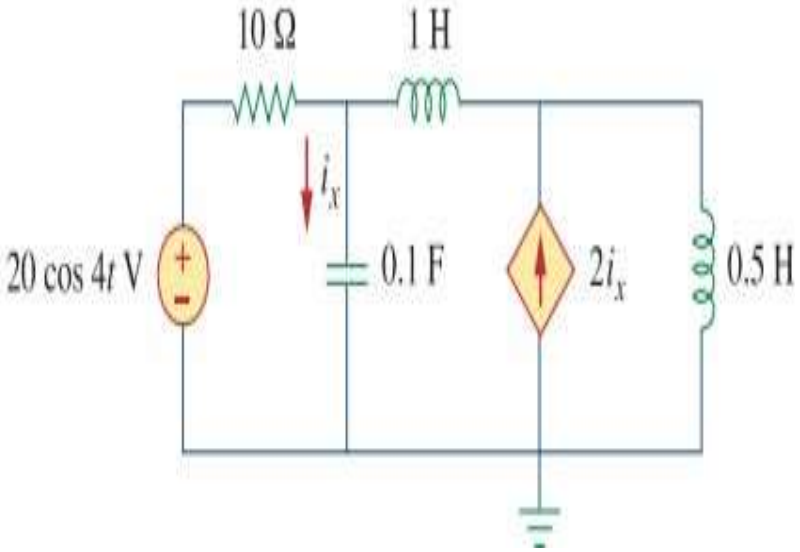


Fig: 10.1

Solution:

We first convert the circuit to the frequency domain:

$$
\begin{aligned}
20\cos 4t &\quad\Rightarrow\quad 20\angle 0^\circ, \qquad \omega = 4 \text{ rad/s} \\
1 \text{ H} &\quad\Rightarrow\quad j\omega L = j4 \\
0.5 \text{ H} &\quad\Rightarrow\quad j\omega L = j2 \\
0.1 \text{ F} &\quad\Rightarrow\quad \frac{1}{j\omega C} = -j2.5
\end{aligned}
$$

Thus, the frequency domain equivalent circuit is as shown in Fig. 10.2.

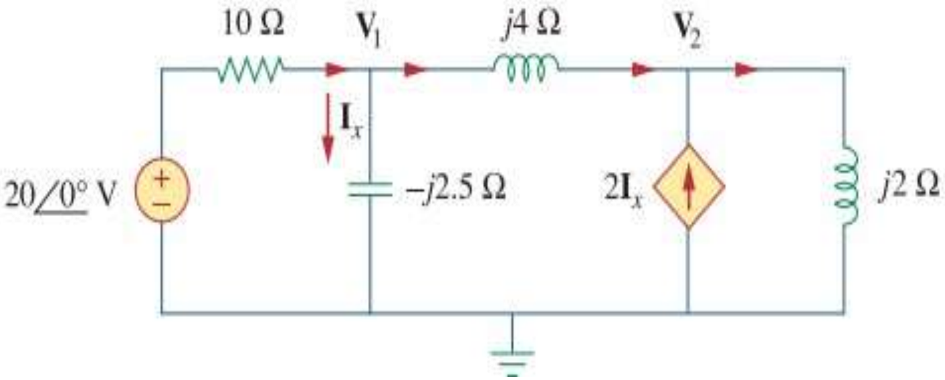


Fig: 10.2

Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20 \qquad \textbf{(10.1.1)}$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we get

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0 \qquad \textbf{(10.1.2)}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\underline{/18.43^\circ} \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\underline{/198.3^\circ} \text{ V}$$

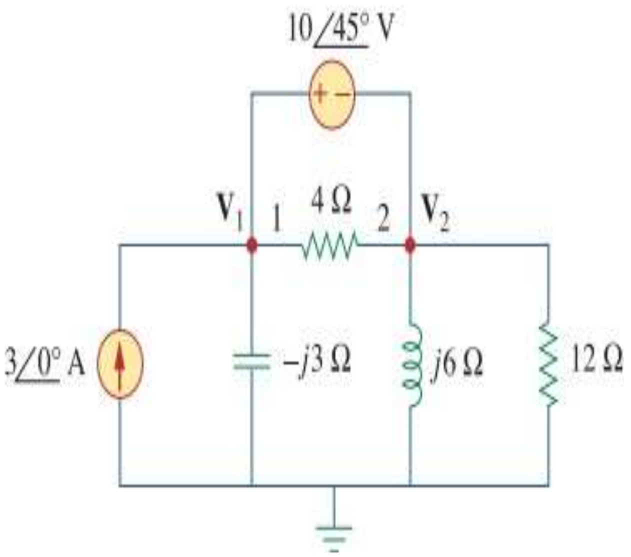
The current $\mathbf{I}_x$ is given by

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\underline{/18.43^\circ}}{2.5\underline{/-90^\circ}} = 7.59\underline{/108.4^\circ} \text{ A}$$

Transforming this to the time domain,

$$i_x = 7.59 \cos(4t + 108.4^\circ) \text{ A}$$

Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit of Fig. 10.4.



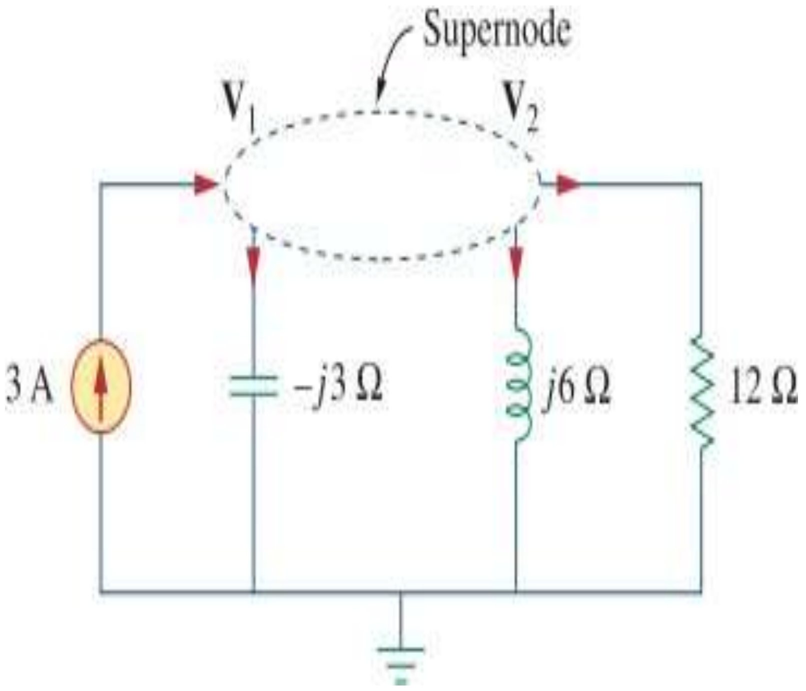
Solution:

Nodes 1 and 2 form a supernode as shown in Fig. 10.5. Applying KCL at the supernode gives

$$3 = \frac{\mathbf{V}_1}{-j3} + \frac{\mathbf{V}_2}{j6} + \frac{\mathbf{V}_2}{12}$$

or

$$36 = j4\mathbf{V}_1 + (1 - j2)\mathbf{V}_2 \qquad \textbf{(10.2.1)}$$



But a voltage source is connected between nodes 1 and 2, so that

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\underline{/45^\circ} \qquad \textbf{(10.2.2)}$$

Substituting Eq. (10.2.2) in Eq. (10.2.1) results in

$$36 - 40\underline{/135^\circ} = (1 + j2)\mathbf{V}_2 \quad \Rightarrow \quad \mathbf{V}_2 = 31.41\underline{/-87.18^\circ} \text{ V}$$

From Eq. (10.2.2),

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\underline{/45^\circ} = 25.78\underline{/-70.48^\circ} \text{ V}$$

Homework:

Calculate $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit shown in Fig. 10.6.

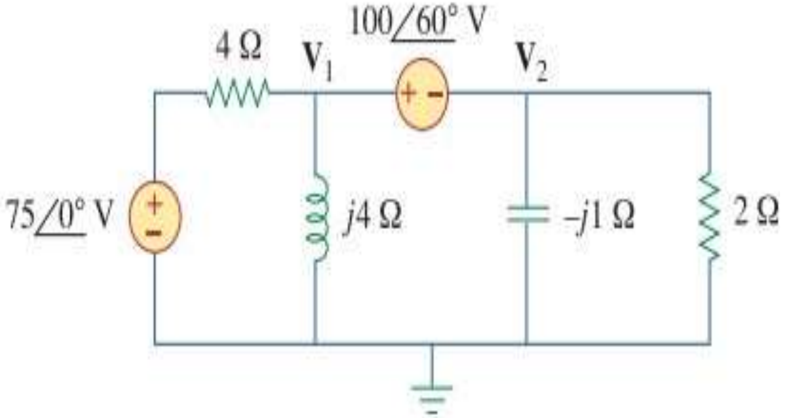


Figure 10.6
For Practice Prob. 10.2.

Answer: $\mathbf{V}_1 = 96.8\angle 69.66^\circ$ V, $\mathbf{V}_2 = 16.88\angle 165.72^\circ$ V.

WEEK-09
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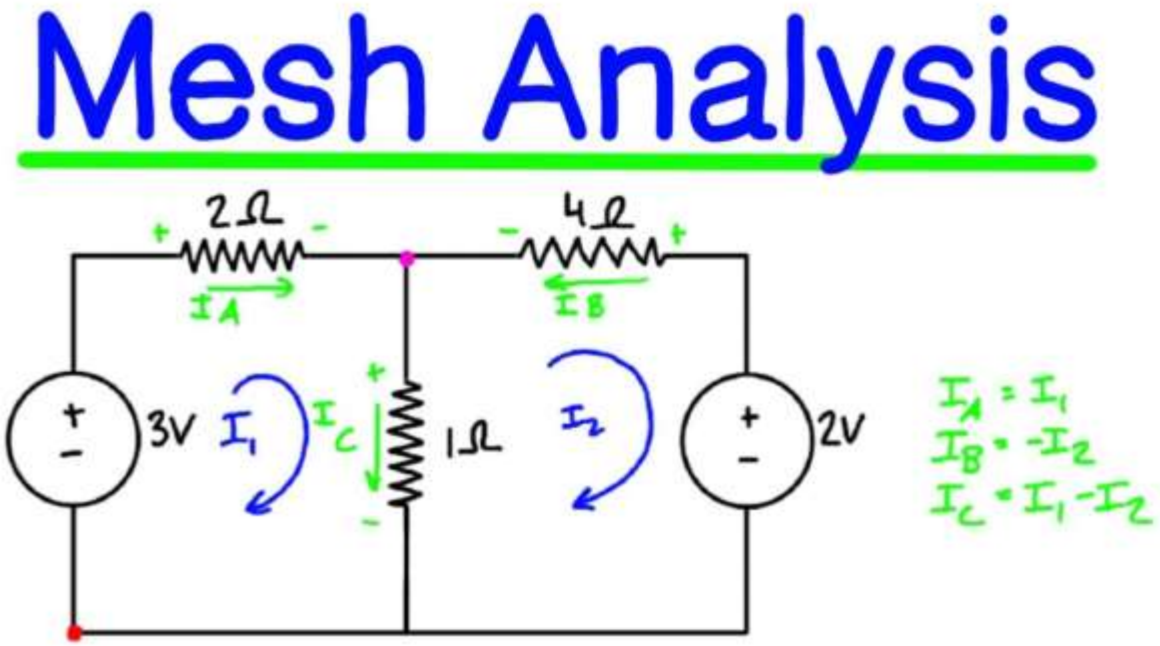
Mesh analysis in AC circuits



What is Mesh Analysis?

What is Mesh Analysis?

Mesh analysis is defined as the method in which the current flowing through a planar circuit is calculated.

A planar circuit is defined as the circuits that are drawn on the plane surface in which there are no wires crossing each other. Therefore, a mesh analysis can also be known as loop analysis or mesh-current method.

Procedure of Mesh Analysis

Step 1:

To identify the meshes and label these mesh currents in either clockwise or counterclockwise direction.

Step 2:

To observe the amount of current that flows through each element in terms of mesh current.

Step 3:

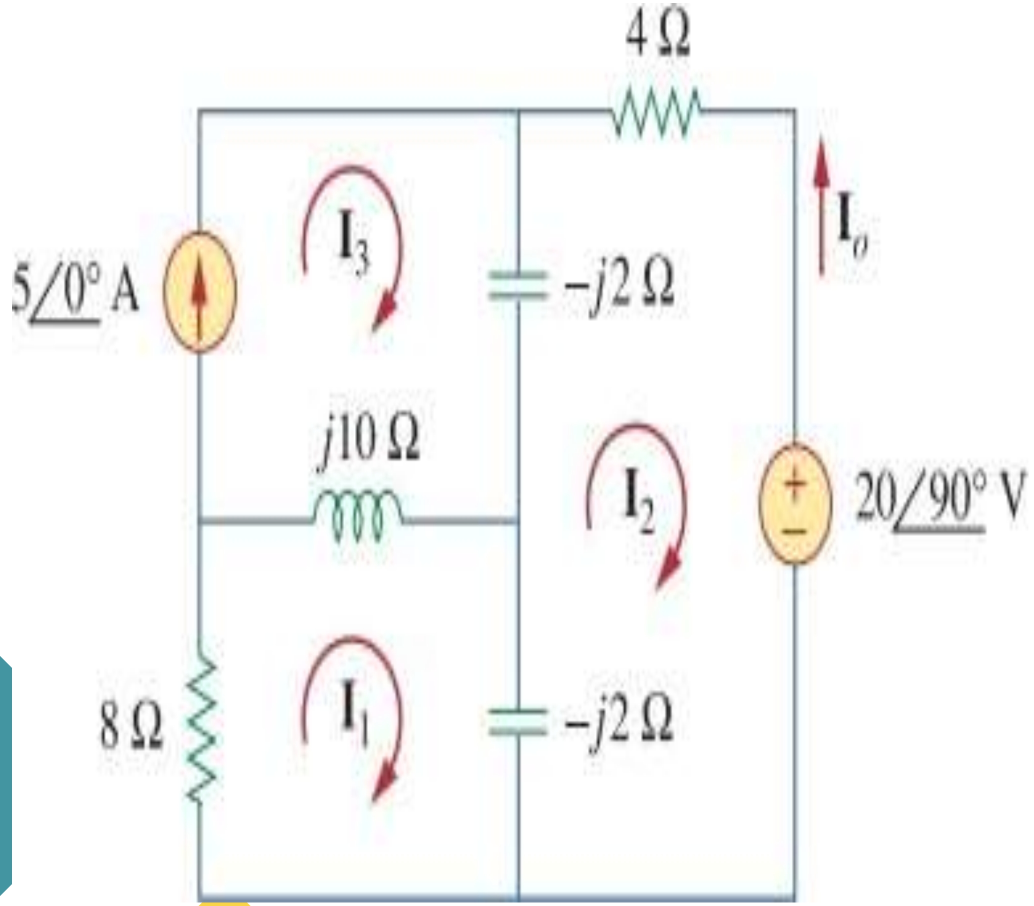
Writing the mesh equations to all meshes using Kirchhoff's voltage law and then Ohm's law.

Step 4:

The mesh currents are obtained by following Step 3 in which the mesh equations are solved.

Mesh Analysis

Find Io of the following circuit using mesh analysis



Solution:

Applying KVL to mesh 1, we obtain

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0 \qquad \textbf{(10.3.1)}$$

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\underline{/90°} = 0 \qquad \textbf{(10.3.2)}$$

For mesh 3, $\mathbf{I}_3 = 5$. Substituting this in Eqs. (10.3.1) and (10.3.2), we get

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50 \qquad \textbf{(10.3.3)}$$

$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10 \qquad \textbf{(10.3.4)}$$

from which we obtain the determinants

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\underline{/-35.22°}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\underline{/-35.22°}}{68} = 6.12\underline{/-35.22°}\text{ A}$$

The desired current is

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\underline{/144.78°}\text{ A}$$

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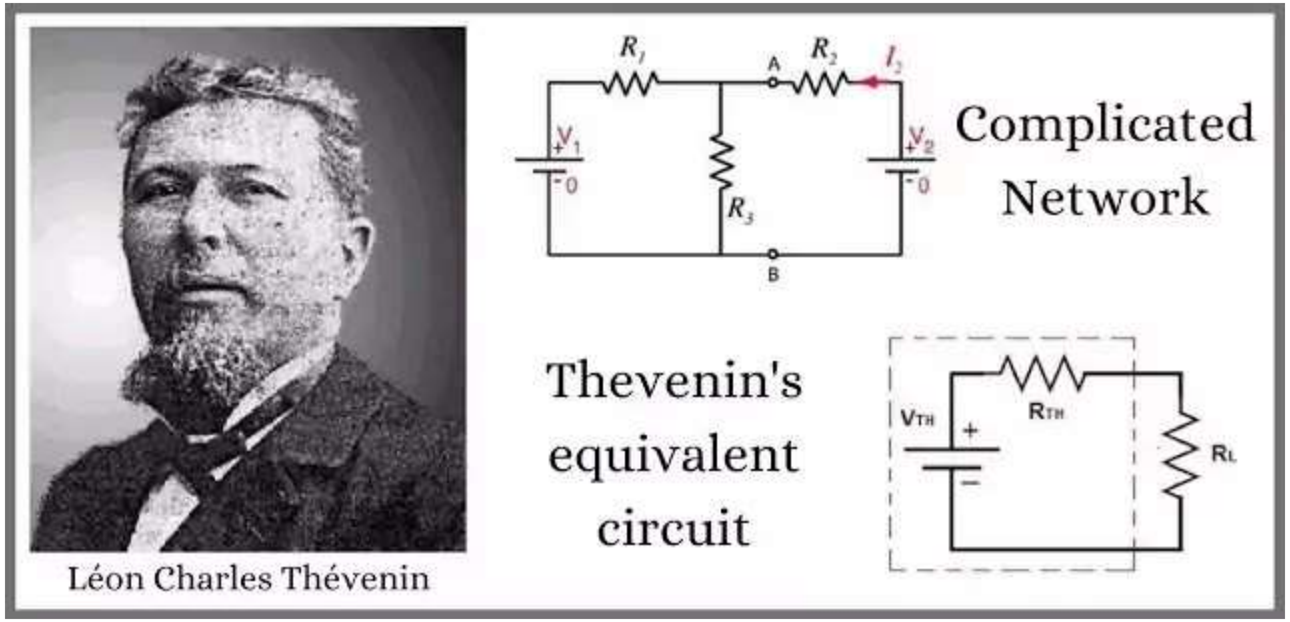
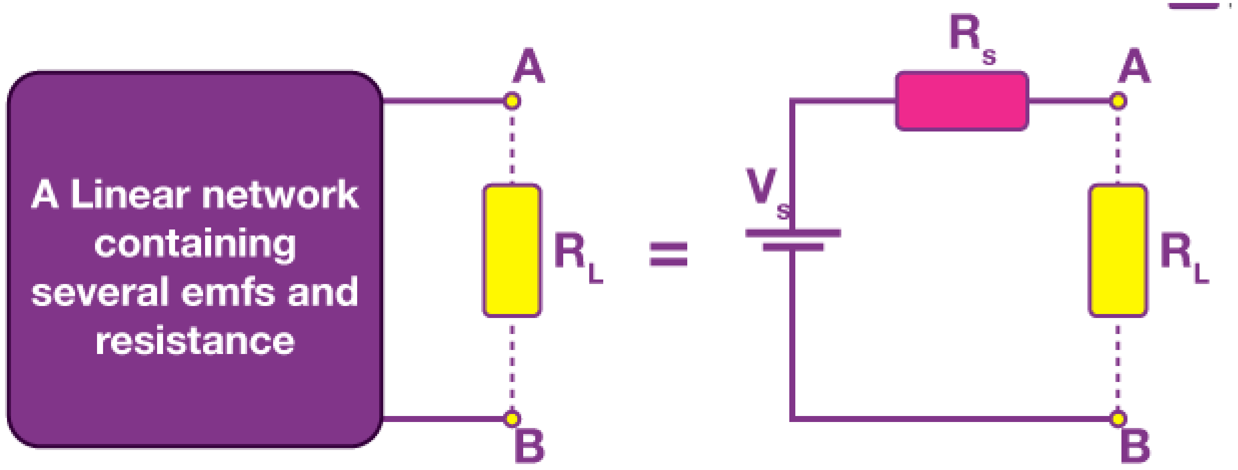
Thevenin Theorem



Thevenin's Theorem

Thevenin’s theorem states that it is possible to simplify any linear circuit, irrespective of how complex it is, to an equivalent circuit with a single voltage source and a series resistance.



Procedure to find Thevenin Circuit

Step 1: For the analysis of the above circuit using Thevenin's theorem, firstly remove the load resistance at the centre.

Step 2: Remove the voltage sources' internal resistance by shorting all the voltage sources connected to the circuit, i.e. v = 0. If current sources are present in the circuit, then remove the internal resistance by open circuiting the sources. This step is done to have an ideal voltage source or an ideal current source for the analysis.

Step 3: Find the equivalent resistance. In the example, the equivalent resistance of the circuit is calculated.

Step 4: Find the equivalent voltage.

Step 5: Draw the Thevenin's equivalent circuit. The Thevenin's equivalent circuit consists of a series resistance and a voltage source.

Thevenin Theorem

Obtain the Thevenin equivalent at terminals *a-b* of the circuit in Fig. 10.22.

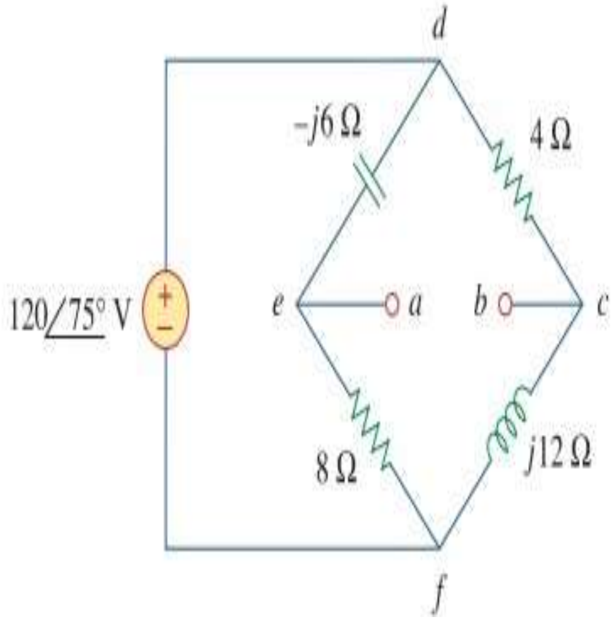
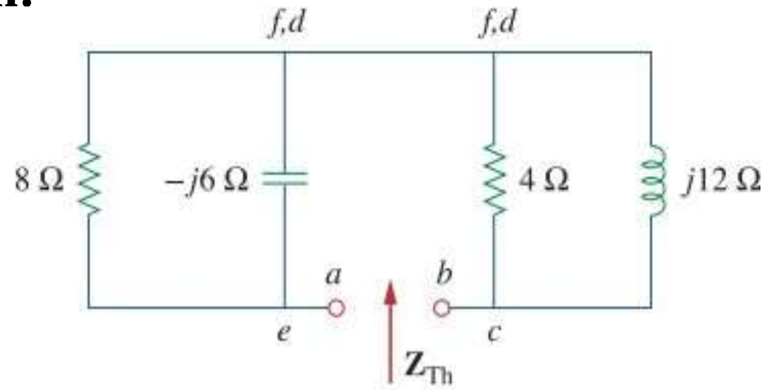


Fig: 10.22

Solution:



(a)

We find $\mathbf{Z}_{Th}$ by setting the voltage source to zero. As shown in Fig. 10.23(a), the 8-Ω resistance is now in parallel with the $-j6$ reactance, so that their combination gives

$$\mathbf{Z}_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3.84\ \Omega$$

Similarly, the 4-Ω resistance is in parallel with the $j12$ reactance, and their combination gives

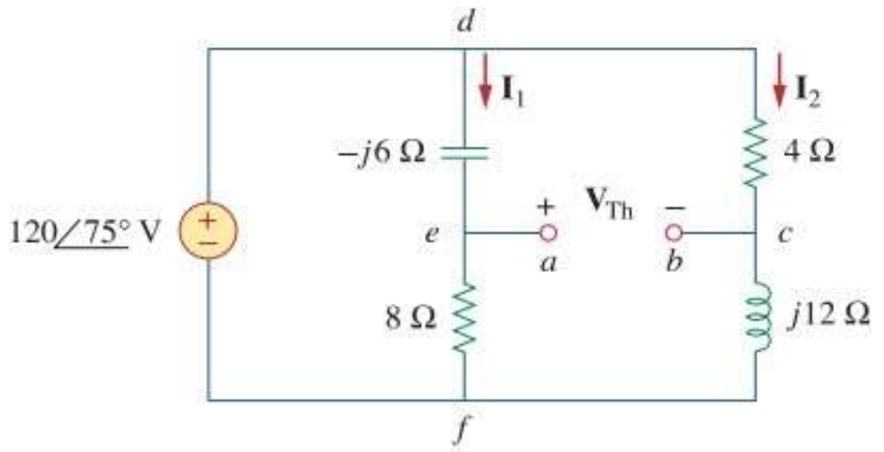
$$\mathbf{Z}_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2\ \Omega$$

The Thevenin impedance is the series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$; that is,

$$\mathbf{Z}_{Th} = \mathbf{Z}_1 + \mathbf{Z}_2 = 6.48 - j2.64\ \Omega$$

To find $\mathbf{V}_{Th}$, consider the circuit in Fig. 10.23(b). Currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are obtained as

$$\mathbf{I}_1 = \frac{120\angle 75^\circ}{8 - j6}\ \text{A}, \qquad \mathbf{I}_2 = \frac{120\angle 75^\circ}{4 + j12}\ \text{A}$$



(b)

Applying KVL around loop *bcdeab* in Fig. 10.23(b) gives

$$\mathbf{V}_{Th} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

or

$$\begin{aligned}\mathbf{V}_{Th} = 4\mathbf{I}_2 + j6\mathbf{I}_1 &= \frac{480\angle 75^\circ}{4 + j12} + \frac{720\angle 75^\circ + 90^\circ}{8 - j6} \\ &= 37.95\angle 3.43^\circ + 72\angle 201.87^\circ \\ &= -28.936 - j24.55 = 37.95\angle 220.31^\circ\ \text{V}\end{aligned}$$

Homework:

Practice Problem 10.8 Find the Thevenin equivalent at terminals *a*-*b* of the circuit in Fig. 10.24.

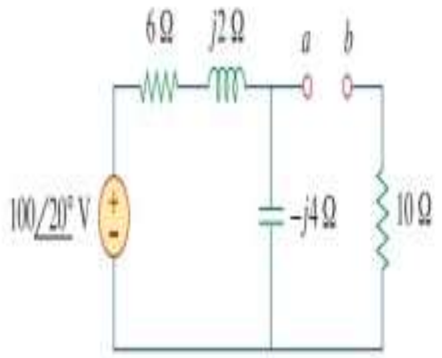


Figure 10.24
For Practice Prob. 10.8.

Answer: $\mathbf{Z}_{Th} = 12.4 - j3.2\ \Omega$, $\mathbf{V}_{Th} = 63.24\angle -51.57^\circ$ V.

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Maximum Power Transfer Theory

Maximum Power Transfer

Maximum Power Transfer Theorem explains that to generate maximum external power through a finite internal resistance (DC network), the resistance of the given load must be equal to the resistance of the available source.

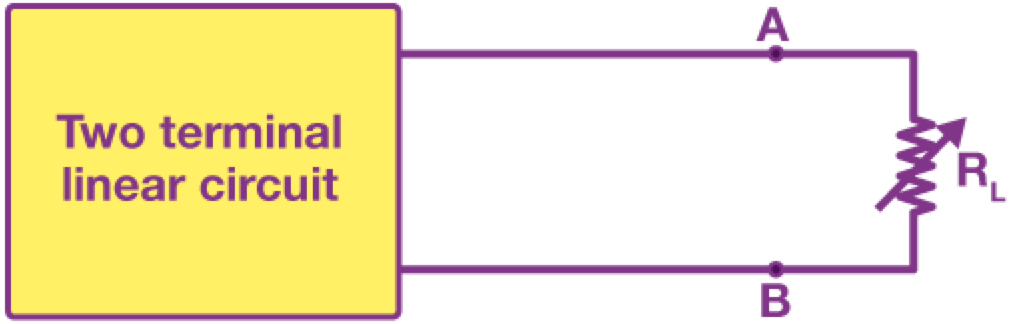
In other words, the resistance of the load must be the same as Thevenin's equivalent resistance.

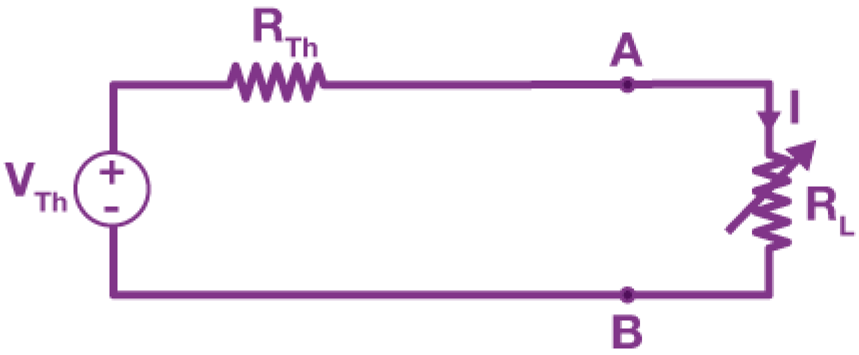
In the case of AC voltage sources, maximum power is produced only if the load impedance's value is equivalent to the complex conjugate of the source impedance.

Maximum Power Transfer

As shown in the figure, a dc source network is connected with variable resistance R_L.

The fundamental Maximum Power Transfer Formula is





Maximum Power Transfer

Example 11.5

Determine the load impedance $\mathbf{Z}_L$ that maximizes the average power drawn from the circuit of Fig. 11.8. What is the maximum average power?

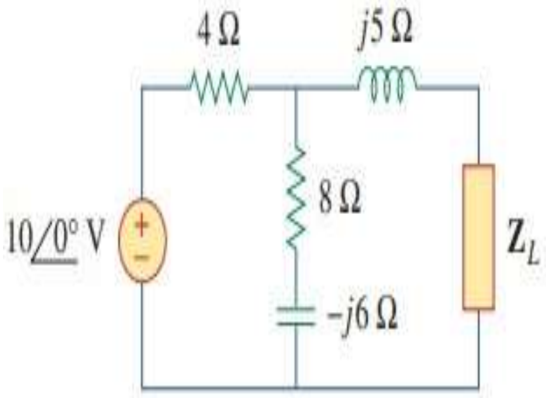
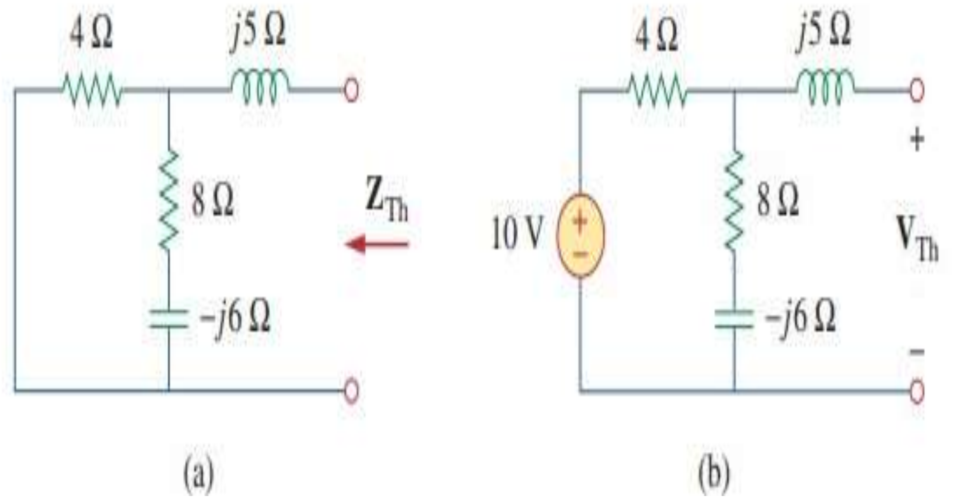


Figure 11.8
For Example 11.5.

Solution:

First we obtain the Thevenin equivalent at the load terminals. To get $\mathbf{Z}_{Th}$, consider the circuit shown in Fig. 11.9(a). We find

$$\mathbf{Z}_{Th} = j5 + 4 \parallel (8 - j6) = j5 + \frac{4(8 - j6)}{4 + 8 - j6} = 2.933 + j4.467\ \Omega$$



To find $\mathbf{V}_{Th}$, consider the circuit in Fig. 11.8(b). By voltage division,

$$\mathbf{V}_{Th} = \frac{8 - j6}{4 + 8 - j6}(10) = 7.454\underline{/-10.3^\circ}\text{ V}$$

The load impedance draws the maximum power from the circuit when

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* = 2.933 - j4.467\ \Omega$$

According to Eq. (11.20), the maximum average power is

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{8R_{Th}} = \frac{(7.454)^2}{8(2.933)} = 2.368\text{ W}$$

Example 11.6

In the circuit in Fig. 11.11, find the value of R_L that will absorb the maximum average power. Calculate that power.

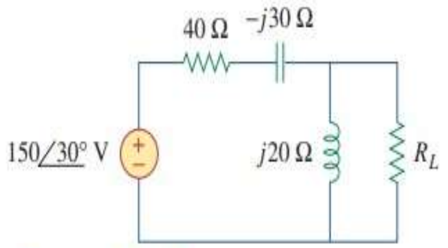


Figure 11.11
For Example 11.6.

Solution:

We first find the Thevenin equivalent at the terminals of R_L.

$$\mathbf{Z}_{Th} = (40 - j30) \parallel j20 = \frac{j20(40 - j30)}{j20 + 40 - j30} = 9.412 + j22.35\ \Omega$$

By voltage division,

$$\mathbf{V}_{Th} = \frac{j20}{j20 + 40 - j30}(150\underline{/30^\circ}) = 72.76\underline{/134^\circ}\text{ V}$$

The value of R_L that will absorb the maximum average power is

$$R_L = |\mathbf{Z}_{Th}| = \sqrt{9.412^2 + 22.35^2} = 24.25\ \Omega$$

The current through the load is

$$\mathbf{I} = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + R_L} = \frac{72.76\underline{/134^\circ}}{33.66 + j22.35} = 1.8\underline{/100.42^\circ}\text{ A}$$

The maximum average power absorbed by R_L is

$$P_{max} = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{1}{2}(1.8)^2(24.25) = 39.29\text{ W}$$

Homework:

Practice Problem 11.5

For the circuit shown in Fig. 11.10, find the load impedance $\mathbf{Z}_L$ that absorbs the maximum average power. Calculate that maximum average power.

Answer: $3.415 - j0.7317\ \Omega$, 51.47 W.

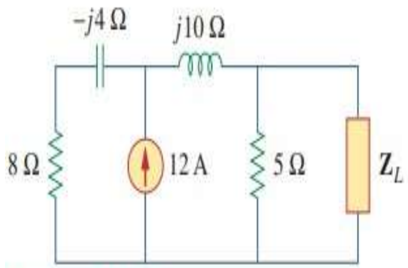


Figure 11.10

Practice Problem 11.6

In Fig. 11.12, the resistor R_L is adjusted until it absorbs the maximum average power. Calculate R_L and the maximum average power absorbed by it.

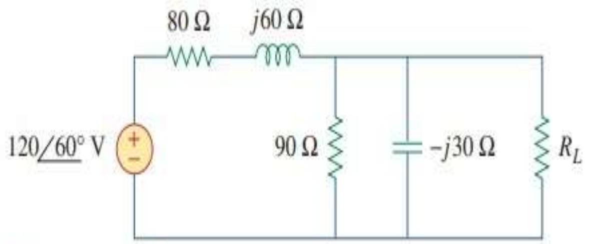


Figure 11.12

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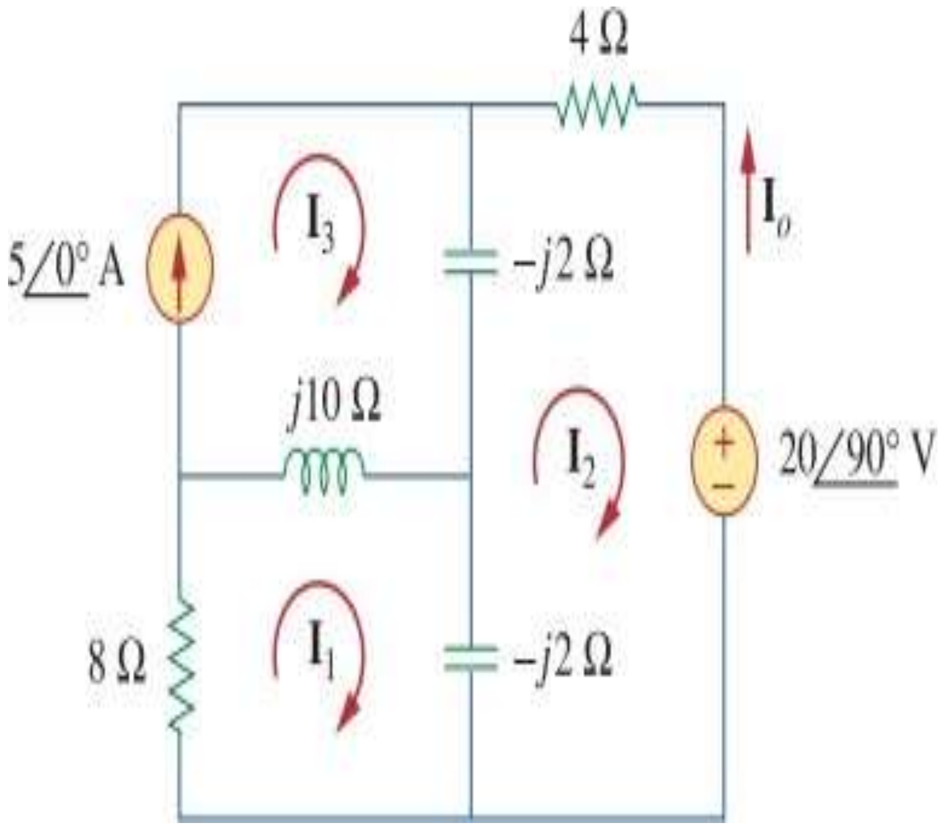
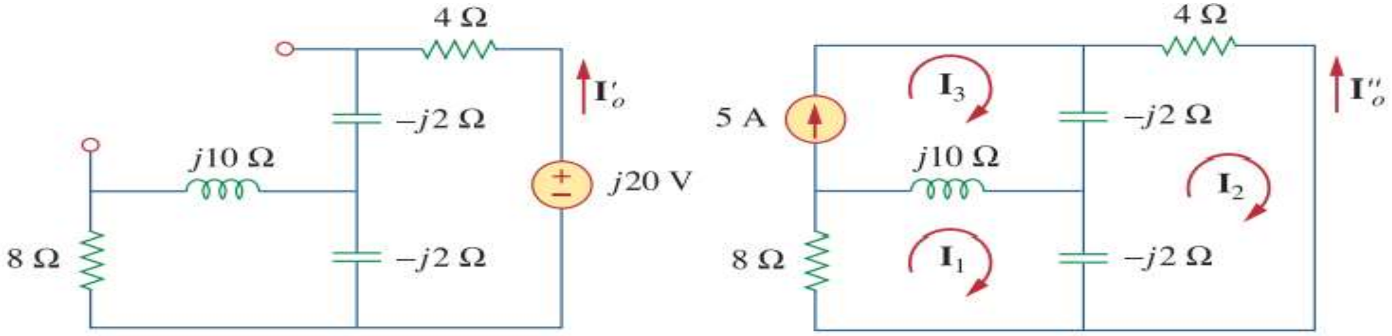
Superposition Theorem

Superposition Theorem

Find Io using superposition theory of the following circuit



Solution:



(a) (b)

Let

$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o \tag{10.5.1}$$

where $\mathbf{I}'_o$ and $\mathbf{I}''_o$ are due to the voltage and current sources, respectively. To find $\mathbf{I}'_o$, consider the circuit in Fig. 10.12(a). If we let $\mathbf{Z}$ be the parallel combination of $-j2$ and $8 + j10$, then

$$\mathbf{Z} = \frac{-j2(8 + j10)}{-2j + 8 + j10} = 0.25 - j2.25$$

and current $\mathbf{I}'_o$ is

$$\mathbf{I}'_o = \frac{j20}{4 - j2 + \mathbf{Z}} = \frac{j20}{4.25 - j4.25}$$

or

$$\mathbf{I}'_o = -2.353 + j2.353 \tag{10.5.2}$$

To get $\mathbf{I}''_o$, consider the circuit in Fig. 10.12(b). For mesh 1,

$$(8 + j8)\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_2 = 0 \tag{10.5.3}$$

For mesh 2,

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j2\mathbf{I}_3 = 0 \tag{10.5.4}$$

For mesh 3,

$$\mathbf{I}_3 = 5 \tag{10.5.5}$$

From Eqs. (10.5.4) and (10.5.5),

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j10 = 0$$

Expressing $\mathbf{I}_1$ in terms of $\mathbf{I}_2$ gives

$$\mathbf{I}_1 = (2 + j2)\mathbf{I}_2 - 5 \tag{10.5.6}$$

Substituting Eqs. (10.5.5) and (10.5.6) into Eq. (10.5.3), we get

$$(8 + j8)[(2 + j2)\mathbf{I}_2 - 5] - j50 + j2\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_2 = \frac{90 - j40}{34} = 2.647 - j1.176$$

Current $\mathbf{I}_o''$ is obtained as

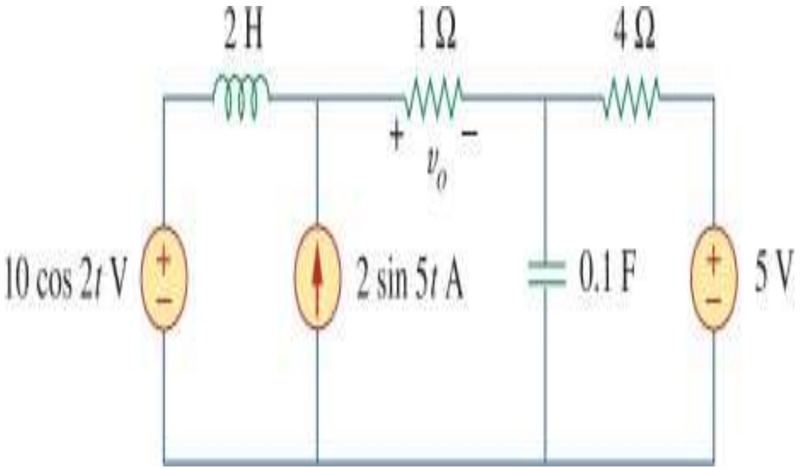
$$\mathbf{I}_o'' = -\mathbf{I}_2 = -2.647 + j1.176 \tag{10.5.7}$$

From Eqs. (10.5.2) and (10.5.7), we write

$$\mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o'' = -5 + j3.529 = 6.12\underline{/144.78^\circ} \text{ A}$$

Homework:

Find v_o of the following circuit using superposition theorem.



Source Transformation:

Calculate $\mathbf{V}_x$ in the circuit of Fig. 10.17 using the method of source transformation.

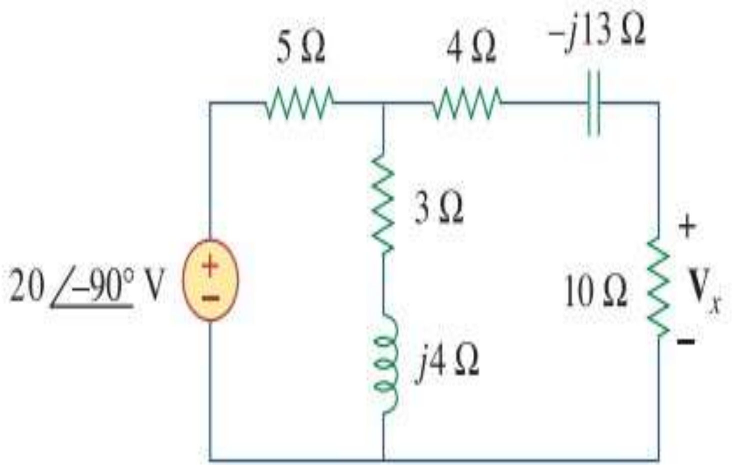


Fig: 10.17

Solution

We transform the voltage source to a current source and obtain the circuit in Fig. 10.18(a), where

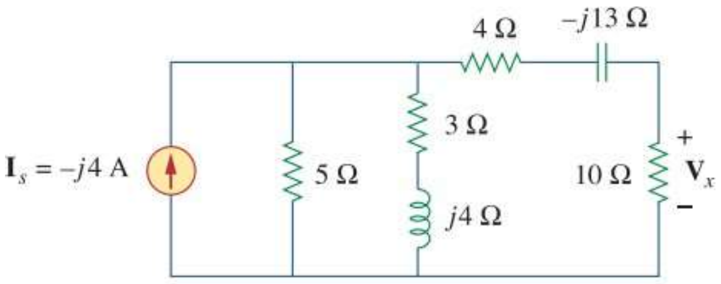
$$\mathbf{I}_s = \frac{20\angle -90^\circ}{5} = 4\angle -90^\circ = -j4 \text{ A}$$

The parallel combination of 5-Ω resistance and $(3 + j4)$ impedance gives

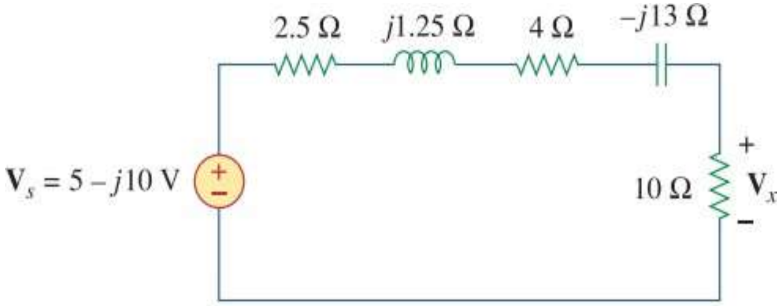
$$\mathbf{Z}_1 = \frac{5(3 + j4)}{8 + j4} = 2.5 + j1.25\ \Omega$$

Converting the current source to a voltage source yields the circuit in Fig. 10.18(b), where

$$\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_1 = -j4(2.5 + j1.25) = 5 - j10 \text{ V}$$



(a)



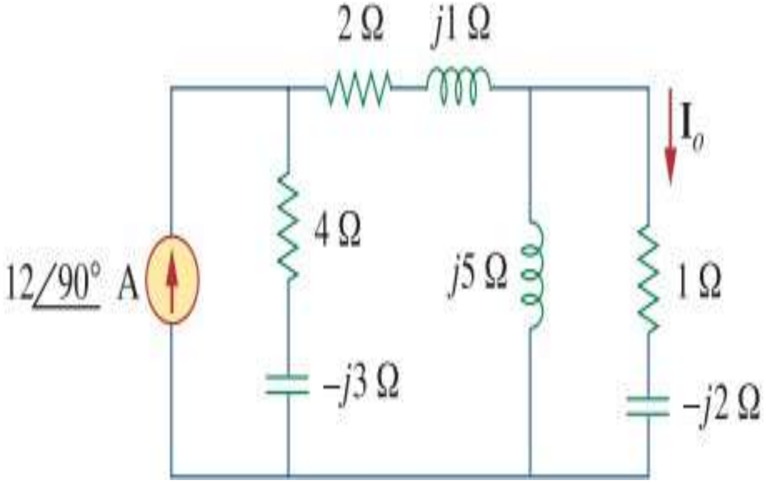
(b)

By voltage division,

$$\mathbf{V}_x = \frac{10}{10 + 2.5 + j1.25 + 4 - j13}(5 - j10) = 5.519\underline{/-28^\circ} \text{ V}$$

Homework:

Find $\mathbf{I}_o$ in the circuit of Fig. 10.19 using the concept of source transformation.



Ans: 9.863< 99.46⁰ A

Given that

$$v(t) = 120\cos(377t + 45^\circ)\text{ V} \quad \text{and} \quad i(t) = 10\cos(377t - 10^\circ)\text{ A}$$

find the instantaneous power and the average power absorbed by the passive linear network of Fig. 11.1.

Solution:

The instantaneous power is given by

$$p = vi = 1200\cos(377t + 45^\circ)\cos(377t - 10^\circ)$$

Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

gives

$$p = 600[\cos(754t + 35^\circ) + \cos 55^\circ]$$

or

$$p(t) = 344.2 + 600\cos(754t + 35^\circ)\text{ W}$$

The average power is

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}120(10)\cos[45^\circ - (-10^\circ)]$$

$$= 600\cos 55^\circ = 344.2\text{ W}$$

which is the constant part of $p(t)$ above.

Calculate the average power absorbed by an impedance $\mathbf{Z} = 30 - j70\,\Omega$ when a voltage $\mathbf{V} = 120\angle 0^\circ$ is applied across it.

Solution:

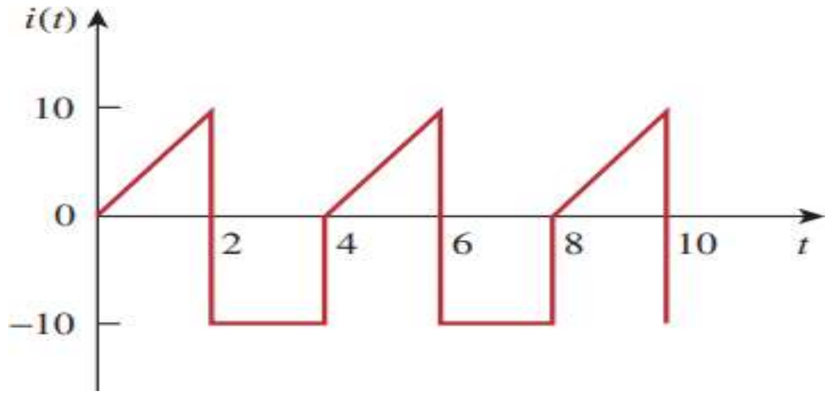
The current through the impedance is

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{120\angle 0^\circ}{30 - j70} = \frac{120\angle 0^\circ}{76.16\angle -66.8^\circ} = 1.576\angle 66.8^\circ\text{ A}$$

The average power is

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120)(1.576)\cos(0 - 66.8^\circ) = 37.24\text{ W}$$

Determine the rms value of the current waveform in Fig. 11.14. If the current is passed through a 2-Ω resistor, find the average power absorbed by the resistor.



Solution:

The period of the waveform is $T = 4$. Over a period, we can write the current waveform as

$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

The rms value is

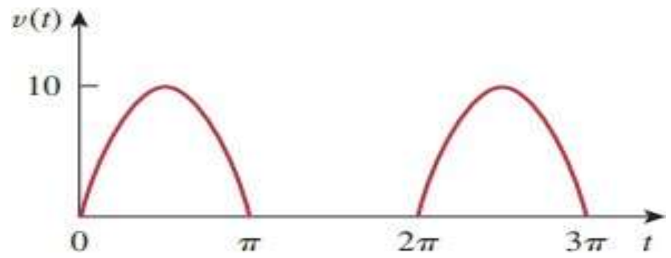
$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2\,dt + \int_2^4 (-10)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{4}\left[25\frac{t^3}{3}\bigg|_0^2 + 100t\bigg|_2^4\right]} = \sqrt{\frac{1}{4}\left(\frac{200}{3} + 200\right)} = 8.165\text{ A}$$

The power absorbed by a 2-Ω resistor is

$$P = I_{\text{rms}}^2 R = (8.165)^2(2) = 133.3\text{ W}$$

The waveform shown in Fig. 11.16 is a half-wave rectified sine wave. Find the rms value and the amount of average power dissipated in a 10-Ω resistor.



Solution:

The period of the voltage waveform is $T = 2\pi$, and

$$v(t) = \begin{cases} 10 \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

The rms value is obtained as

$$V_{\text{rms}}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt = \frac{1}{2\pi}\left[\int_0^{\pi} (10 \sin t)^2\,dt + \int_{\pi}^{2\pi} 0^2\,dt\right]$$

But $\sin^2 t = \frac{1}{2}(1 - \cos 2t)$. Hence,

$$V_{\text{rms}}^2 = \frac{1}{2\pi}\int_0^{\pi} \frac{100}{2}(1 - \cos 2t)\,dt = \frac{50}{2\pi}\left(t - \frac{\sin 2t}{2}\right)\Big|_0^{\pi}$$

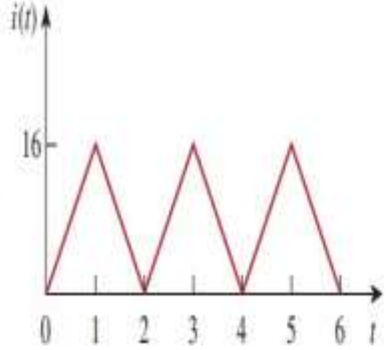
$$= \frac{50}{2\pi}\left(\pi - \frac{1}{2}\sin 2\pi - 0\right) = 25, \qquad V_{\text{rms}} = 5\text{ V}$$

The average power absorbed is

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{5^2}{10} = 2.5\text{ W}$$

Practice Problem 11.7

Find the rms value of the current waveform of Fig. 11.15. If the current flows through a 9-Ω resistor, calculate the average power absorbed by the resistor.



Answer: 9.238 A, 768 W.

A series-connected load draws a current $i(t) = 4\cos(100\pi t + 10^\circ)$ A when the applied voltage is $v(t) = 120\cos(100\pi t - 20^\circ)$ V. Find the apparent power and the power factor of the load. Determine the element values that form the series-connected load.

Solution:

The apparent power is

$$S = V_{\text{rms}} I_{\text{rms}} = \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = 240 \text{ VA}$$

The power factor is

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos(-20^\circ - 10^\circ) = 0.866 \qquad \text{(leading)}$$

The pf is leading because the current leads the voltage. The pf may also be obtained from the load impedance.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120\underline{/-20^\circ}}{4\underline{/10^\circ}} = 30\underline{/-30^\circ} = 25.98 - j15\ \Omega$$

$$\text{pf} = \cos(-30^\circ) = 0.866 \qquad \text{(leading)}$$

The load impedance $\mathbf{Z}$ can be modeled by a 25.98-Ω resistor in series with a capacitor with

$$X_C = -15 = -\frac{1}{\omega C}$$

or

$$C = \frac{1}{15\omega} = \frac{1}{15 \times 100\pi} = 212.2\ \mu\text{F}$$

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3-Phase System

Problems solution on 3-phase AC load

3-phase source and load connection systems

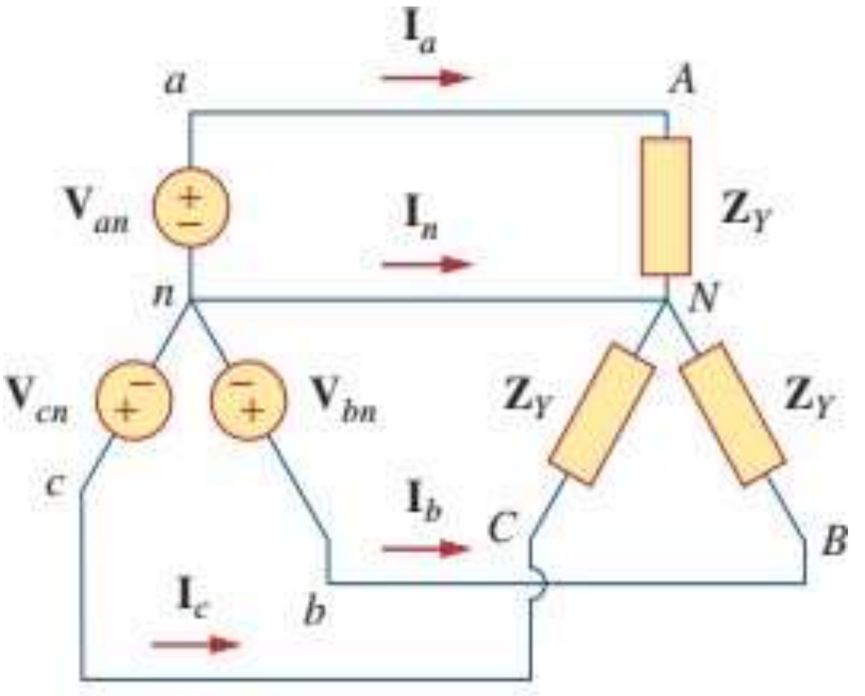


Figure 12.10
Balanced Y-Y connection.

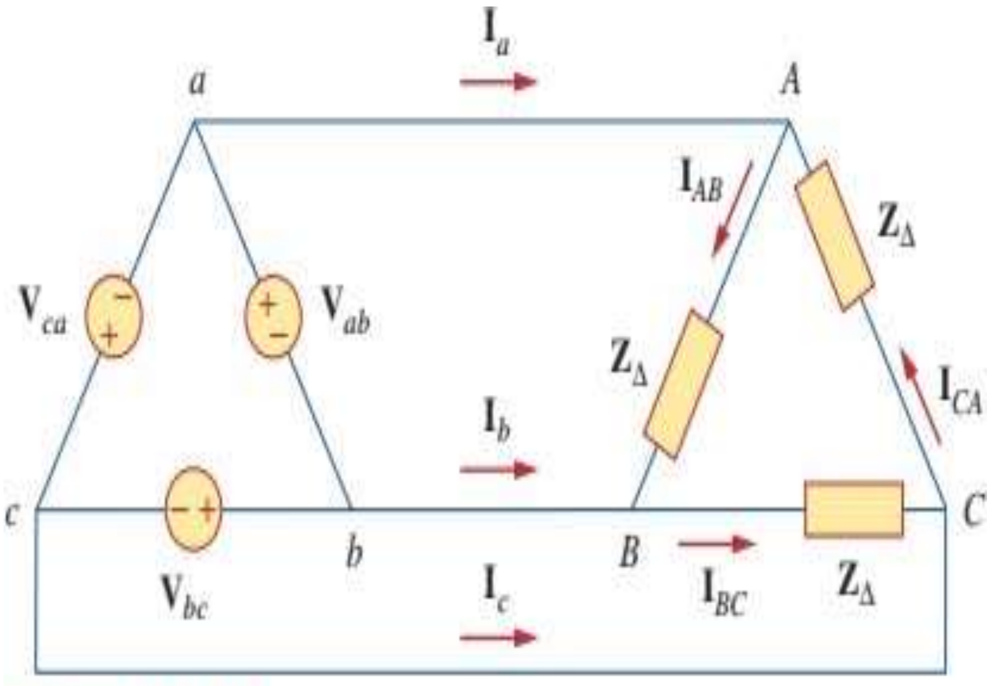


Figure 12.17
A balanced Δ-Δ connection.

Problems solution on 3-phase AC load

3-phase source and load connection systems

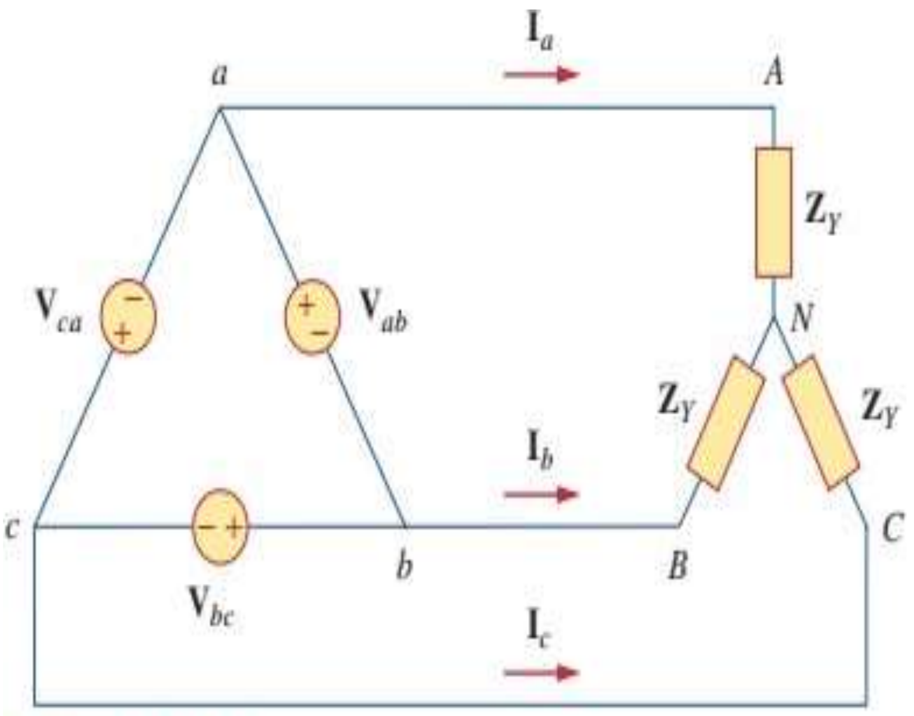


Figure 12.18
A balanced Δ-Y connection.

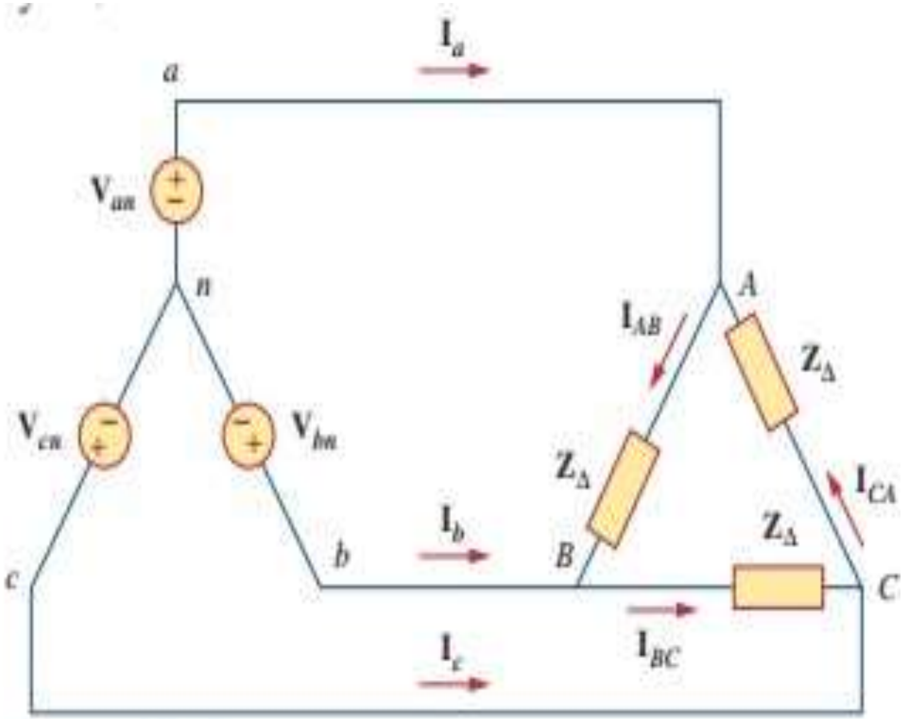
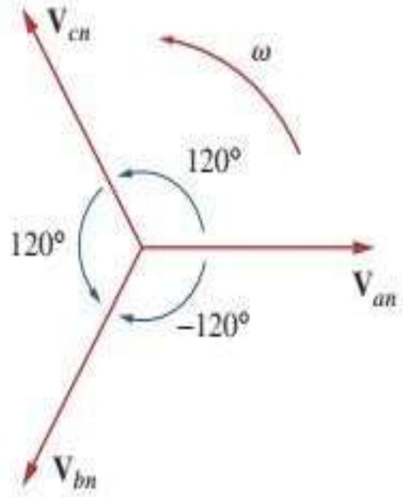
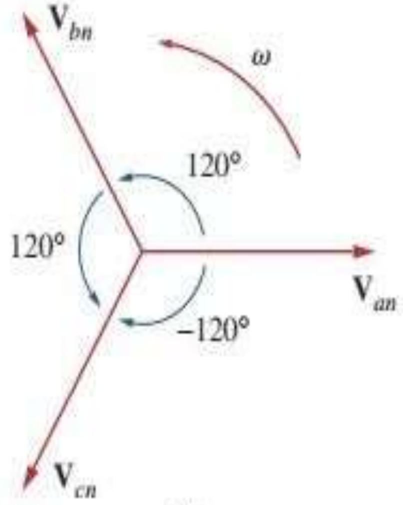


Figure 12.14
Balanced Y-Δ connection.



(a)



(b)

re 12.7

sequences: (a) *abc* or positive nce, (b) *acb* or negative sequence.

Let us consider the wye-connected voltages in Fig. 12.6(a) for now. The voltages $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, and $\mathbf{V}_{cn}$ are respectively between lines *a*, *b*, and *c*, and the neutral line *n*. These voltages are called *phase voltages*. If the voltage sources have the same amplitude and frequency ω and are out of phase with each other by 120°, the voltages are said to be *balanced*. This implies that

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0 \tag{12.1}$$

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}| \tag{12.2}$$

Thus,

Balanced phase voltages are equal in magnitude and are out of phase with each other by 120°.

Since the three-phase voltages are 120° out of phase with each other, there are two possible combinations. One possibility is shown in Fig. 12.7(a) and expressed mathematically as

$$\boxed{\begin{aligned} \mathbf{V}_{an} &= V_p\underline{/0^\circ} \\ \mathbf{V}_{bn} &= V_p\underline{/-120^\circ} \\ \mathbf{V}_{cn} &= V_p\underline{/-240^\circ} = V_p\underline{/+120^\circ} \end{aligned}} \tag{12.3}$$

Calculate the line currents in the three-wire Y-Y system of Fig. 12.13.

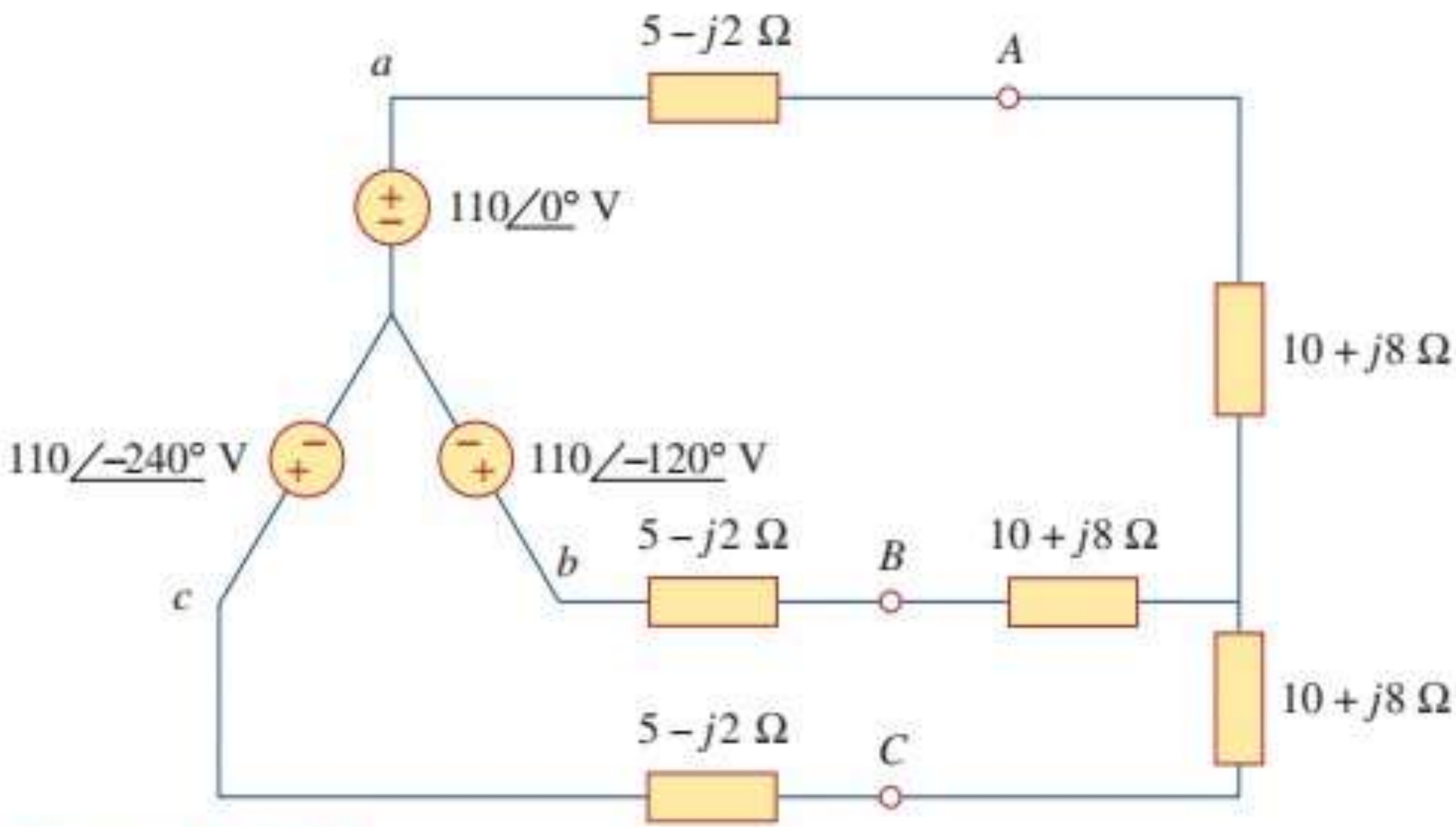


Figure 12.13
Three-wire Y-Y system; for Example 12.2.

Example 12.2

Solution:

The three-phase circuit in Fig. 12.13 is balanced; we may replace it with its single-phase equivalent circuit such as in Fig. 12.12. We obtain $\mathbf{I}_a$ from the single-phase analysis as

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155\underline{/21.8^\circ}$. Hence,

$$\mathbf{I}_a = \frac{110\underline{/0^\circ}}{16.155\underline{/21.8^\circ}} = 6.81\underline{/-21.8^\circ} \text{ A}$$

Since the source voltages in Fig. 12.13 are in positive sequence, the line currents are also in positive sequence:

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 6.81\underline{/-141.8^\circ} \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/-240^\circ} = 6.81\underline{/-261.8^\circ} \text{ A} = 6.81\underline{/98.2^\circ} \text{ A}$$

Example 12.3

A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 100\underline{/10^\circ}$ V is connected to a Δ-connected balanced load $(8 + j4)\ \Omega$ per phase. Calculate the phase and line currents.

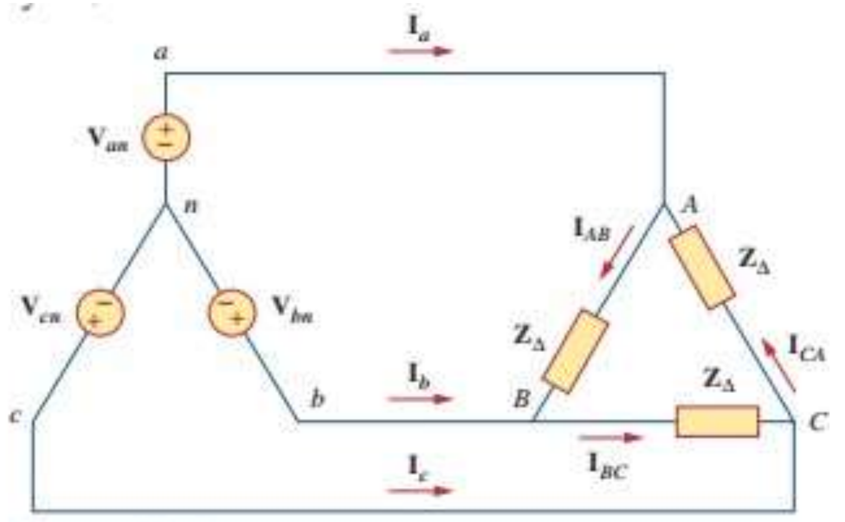


Figure 12.14
Balanced Y-Δ connection.

Solution:

This can be solved in two ways.

METHOD 1 The load impedance is

$$\mathbf{Z}_\Delta = 8 + j4 = 8.944\underline{/26.57^\circ}\ \Omega$$

If the phase voltage $\mathbf{V}_{an} = 100\underline{/10^\circ}$, then the line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\underline{/30^\circ} = 100\sqrt{3}\underline{/10^\circ + 30^\circ} = \mathbf{V}_{AB}$$

or

$$\mathbf{V}_{AB} = 173.2\underline{/40^\circ}\ \text{V}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{173.2\underline{/40^\circ}}{8.944\underline{/26.57^\circ}} = 19.36\underline{/13.43^\circ}\ \text{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\underline{/-120^\circ} = 19.36\underline{/-106.57^\circ}\ \text{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/+120^\circ} = 19.36\underline{/133.43^\circ}\ \text{A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ} = \sqrt{3}(19.36)\underline{/13.43^\circ - 30^\circ}$$

$$= 33.53\underline{/-16.57^\circ}\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 33.53\underline{/-136.57^\circ}\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ} = 33.53\underline{/103.43^\circ}\ \text{A}$$

METHOD 2 Alternatively, using single-phase analysis,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\Delta/3} = \frac{100\underline{/10^\circ}}{2.981\underline{/26.57^\circ}} = 33.54\underline{/-16.57^\circ}\ \text{A}$$

as above. Other line currents are obtained using the *abc* phase sequence.

Example 12.9

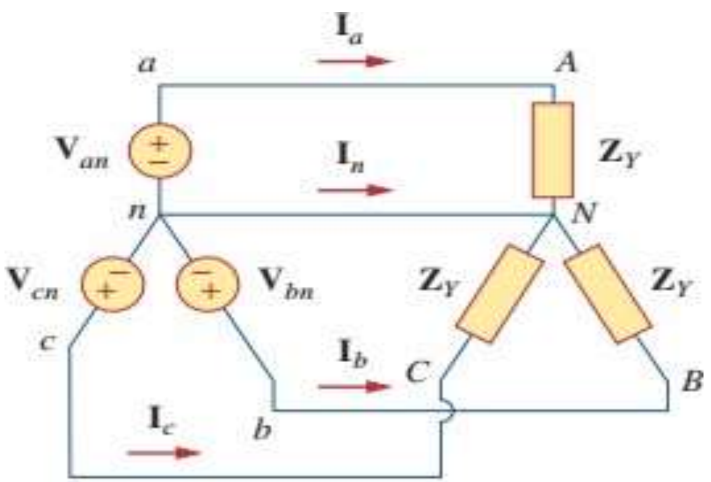


Figure 12.10
Balanced Y-Y connection.

The unbalanced Y-load of Fig. 12.23 has balanced voltages of 100 V and the *acb* sequence. Calculate the line currents and the neutral current. Take $\mathbf{Z}_A = 15\ \Omega$, $\mathbf{Z}_B = 10 + j5\ \Omega$, $\mathbf{Z}_C = 6 - j8\ \Omega$.

Solution:

Using Eq. (12.59), the line currents are

$$\mathbf{I}_a = \frac{100\angle 0^\circ}{15} = 6.67\angle 0^\circ \text{ A}$$

$$\mathbf{I}_b = \frac{100\angle 120^\circ}{10 + j5} = \frac{100\angle 120^\circ}{11.18\angle 26.56^\circ} = 8.94\angle 93.44^\circ \text{ A}$$

$$\mathbf{I}_c = \frac{100\angle -120^\circ}{6 - j8} = \frac{100\angle -120^\circ}{10\angle -53.13^\circ} = 10\angle -66.87^\circ \text{ A}$$

Using Eq. (12.60), the current in the neutral line is

$$\begin{aligned}\mathbf{I}_n &= -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = -(6.67 - 0.54 + j8.92 + 3.93 - j9.2)\\ &= -10.06 + j0.28 = 10.06\angle 178.4^\circ \text{ A}\end{aligned}$$

....
....

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Single Phase and Three Phase Power Measurement

Single Phase & Three-Phase Power Measurement

Section 11.9 presented the wattmeter as the instrument for measuring the average (or real) power in single-phase circuits. A single wattmeter can also measure the average power in a three-phase system that is balanced, so that $P_1 = P_2 = P_3$; the total power is three times the reading of that one wattmeter. However, two or three single-phase wattmeters are necessary to measure power if the system is unbalanced. The *three-wattmeter method* of power measurement, shown in Fig. 12.33, will work regardless of whether the load is balanced or unbalanced, wye- or delta-connected. The three-wattmeter method is well suited for power measurement in a three-phase system where the power factor is constantly changing. The total average power is the algebraic sum of the three wattmeter readings,

$$P_T = P_1 + P_2 + P_3 \tag{12.61}$$

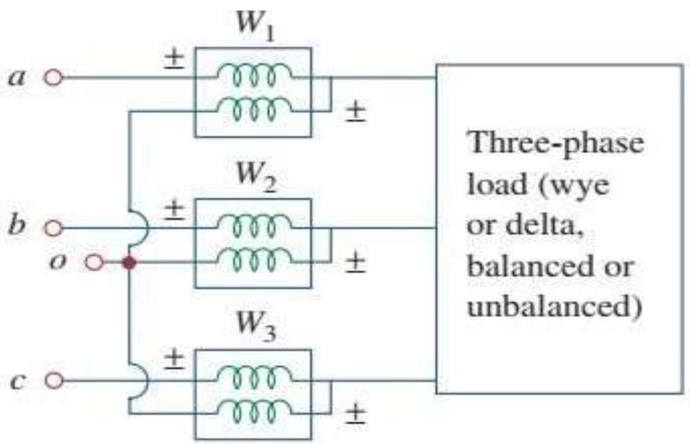

Figure 12.33
Three-wattmeter method for measuring three-phase power.

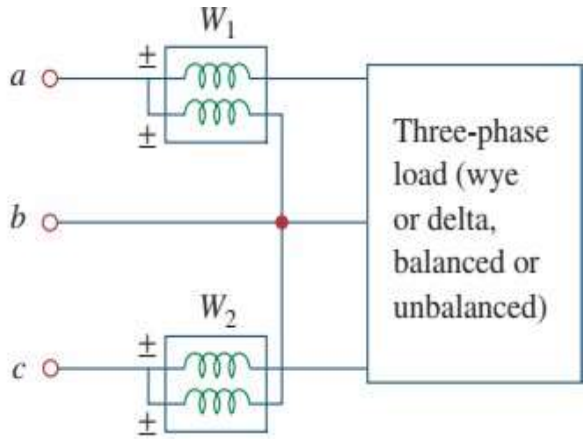

Figure 12.34
Two-wattmeter method for measuring three-phase power.

where P_1, P_2, and P_3 correspond to the readings of wattmeters W_1, W_2, and W_3, respectively. Notice that the common or reference point o in Fig. 12.33 is selected arbitrarily. If the load is wye-connected, point o can be connected to the neutral point n. For a delta-connected load, point o can be connected to any point. If point o is connected to point b, for example, the voltage coil in wattmeter W_2 reads zero and $P_2 = 0$, indicating that wattmeter W_2 is not necessary. Thus, two wattmeters are sufficient to measure the total power.

unbalanced. The total real power is equal to the algebraic sum of the two wattmeter readings,

$$P_T = P_1 + P_2 \tag{12.62}$$

We will show here that the method works for a balanced three-phase system.

The *two-wattmeter method* is the most commonly used method for three-phase power measurement. The two wattmeters must be properly connected to any two phases, as shown typically in Fig. 12.34. Notice that the current coil of each wattmeter measures the line current, while the respective voltage coil is connected between the line and the third line and measures the line voltage. Also notice that the $\pm$ terminal of the voltage coil is connected to the line to which the corresponding current coil is connected. Although the individual wattmeters no longer read the power taken by any particular phase, the algebraic sum of the two wattmeter readings equals the total average power absorbed by the load, regardless of whether it is wye- or delta-connected, balanced or

Consider the balanced, wye-connected load in Fig. 12.35. Our objective is to apply the two-wattmeter method to find the average power absorbed by the load. Assume the source is in the *abc* sequence and the load impedance $\mathbf{Z}_Y = Z_Y \underline{/\theta}$. Due to the load impedance, each voltage coil leads its current coil by θ, so that the power factor is $\cos\theta$. We recall that each line voltage leads the corresponding phase voltage by 30°. Thus, the total phase difference between the phase current $\mathbf{I}_a$ and line voltage $\mathbf{V}_{ab}$ is $\theta + 30°$, and the average power read by wattmeter W_1 is

$$P_1 = \text{Re}[\mathbf{V}_{ab}\mathbf{I}_a^*] = V_{ab} I_a \cos(\theta + 30°) = V_L I_L \cos(\theta + 30°) \tag{12.63}$$

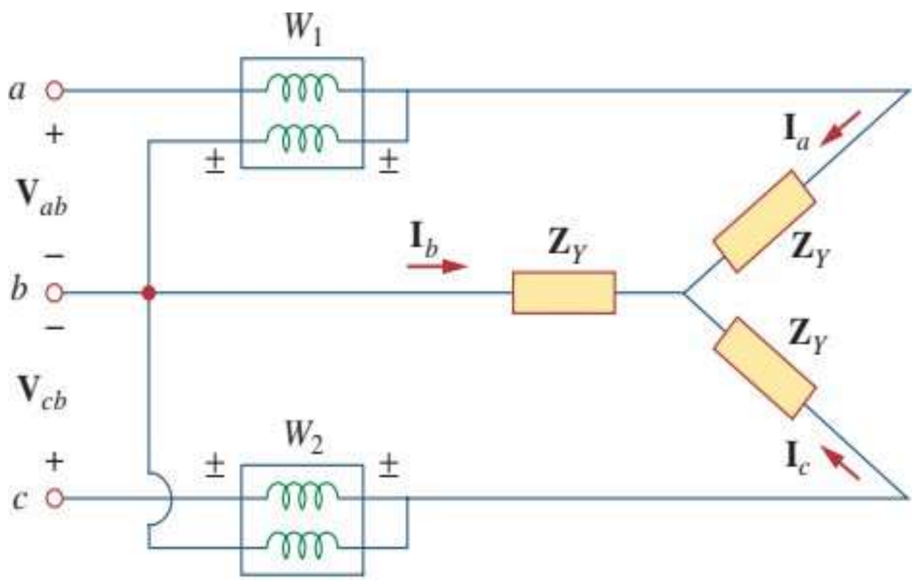

Figure 12.35
Two-wattmeter method applied to a balanced wye load.

Similarly, we can show that the average power read by wattmeter 2 is

$$P_2 = \text{Re}[\mathbf{V}_{cb}\mathbf{I}_c^*] = V_{cb}I_c\cos(\theta - 30^\circ) = V_L I_L \cos(\theta - 30^\circ) \quad \textbf{(12.64)}$$

We now use the trigonometric identities

$$\begin{aligned} \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \end{aligned} \quad \textbf{(12.65)}$$

to find the sum and the difference of the two wattmeter readings in Eqs. (12.63) and (12.64):

$\mathrm{V_{ab}=V_a\text{-}V_b=V_a+(\text{-}V_b)}$

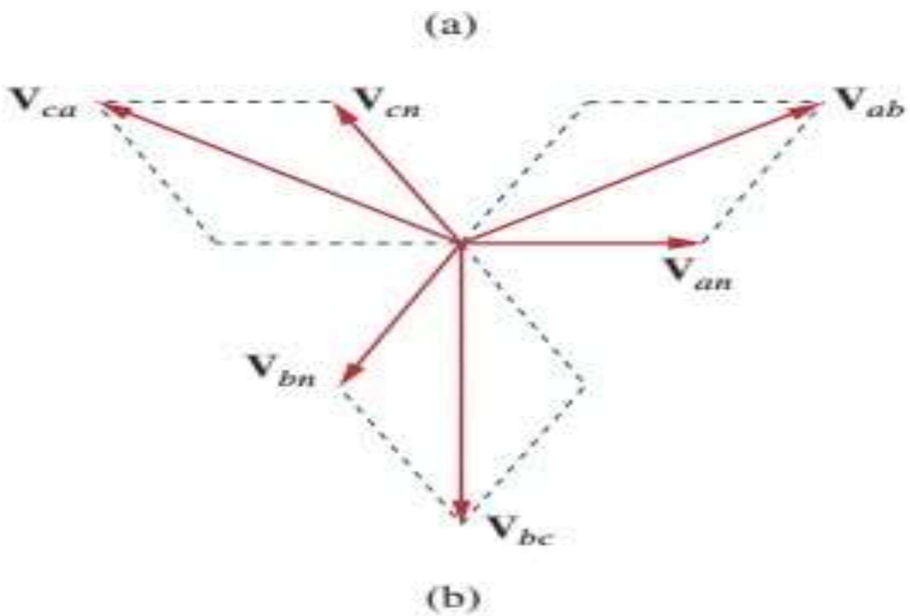

Figure 12.11
Phasor diagrams illustrating the relationship between line voltages and phase voltages.

Similarly,

$$\begin{aligned} P_1 - P_2 &= V_L I_L[\cos(\theta + 30^\circ) - \cos(\theta - 30^\circ)] \\ &= V_l I_L(\cos\theta\cos 30^\circ - \sin\theta \sin 30^\circ \\ &\qquad - \cos\theta \cos 30^\circ - \sin\theta \sin 30^\circ) \\ &= -V_L I_L 2 \sin 30^\circ \sin\theta \end{aligned} \quad \textbf{(12.68)}$$

$$P_2 - P_1 = V_L I_L \sin\theta$$

$$
\begin{aligned}
P_1 + P_2 &= V_L I_L[\cos(\theta + 30°) + \cos(\theta - 30°)] \\
&= V_L I_L(\cos\theta \cos 30° - \sin\theta \sin 30° \\
&\quad + \cos\theta \cos 30° + \sin\theta \sin 30°) \\
&= V_L I_L 2 \cos 30° \cos\theta = \sqrt{3} V_L I_L \cos\theta
\end{aligned} \tag{12.66}
$$

since $2 \cos 30° = \sqrt{3}$. Comparing Eq. (12.66) with Eq. (12.50) shows that the sum of the wattmeter readings gives the total average power,

$$\boxed{P_T = P_1 + P_2} \tag{12.67}$$

since $2 \sin 30° = 1$. Comparing Eq. (12.68) with Eq. (12.51) shows that the difference of the wattmeter readings is proportional to the total reactive power, or

$$\boxed{Q_T = \sqrt{3}(P_2 - P_1)} \tag{12.69}$$

From Eqs. (12.67) and (12.69), the total apparent power can be obtained as

$$S_T = \sqrt{P_T^2 + Q_T^2} \tag{12.70}$$

Dividing Eq. (12.69) by Eq. (12.67) gives the tangent of the power factor angle as

$$\tan\theta = \frac{Q_T}{P_T} = \sqrt{3}\frac{P_2 - P_1}{P_2 + P_1} \tag{12.71}$$

from which we can obtain the power factor as pf $= \cos\theta$. Thus, the two-wattmeter method not only provides the total real and reactive powers, it can also be used to compute the power factor. From Eqs. (12.67), (12.69), and (12.71), we conclude that:

1. If $P_2 = P_1$, the load is resistive.
2. If $P_2 > P_1$, the load is inductive.
3. If $P_2 < P_1$ the load is capacitive.

Although these results are derived from a balanced wye-connected load, they are equally valid for a balanced delta-connected load. However, the two-wattmeter method cannot be used for power measurement in a three-phase four-wire system unless the current through the neutral line is zero. We use the three-wattmeter method to measure the real power in a three-phase four-wire system.

WEEK-15

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Example 12.9

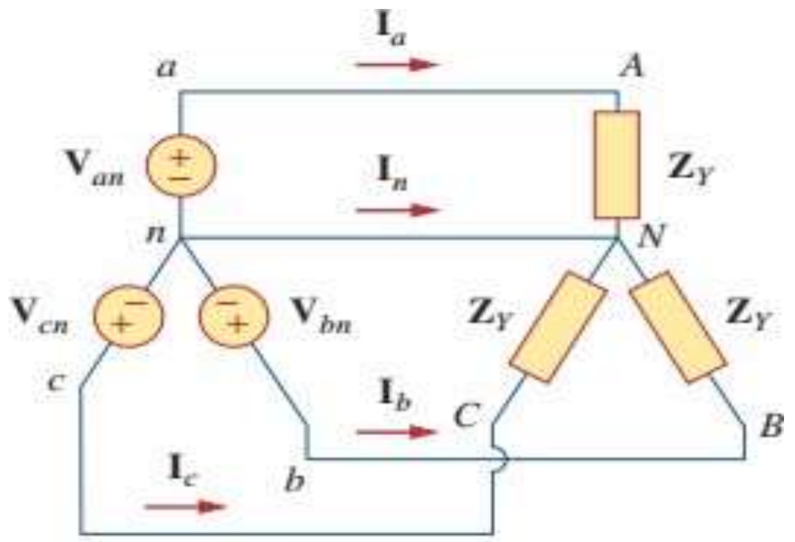

Figure 12.10
Balanced Y-Y connection.

The unbalanced Y-load of Fig. 12.23 has balanced voltages of 100 V and the *acb* sequence. Calculate the line currents and the neutral current. Take $\mathbf{Z}_A = 15\ \Omega$, $\mathbf{Z}_B = 10 + j5\ \Omega$, $\mathbf{Z}_C = 6 - j8\ \Omega$.

Solution:

Using Eq. (12.59), the line currents are

$$\mathbf{I}_a = \frac{100\underline{/0^\circ}}{15} = 6.67\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_b = \frac{100\underline{/120^\circ}}{10 + j5} = \frac{100\underline{/120^\circ}}{11.18\underline{/26.56^\circ}} = 8.94\underline{/93.44^\circ}\text{ A}$$

$$\mathbf{I}_c = \frac{100\underline{/-120^\circ}}{6 - j8} = \frac{100\underline{/-120^\circ}}{10\underline{/-53.13^\circ}} = 10\underline{/-66.87^\circ}\text{ A}$$

Using Eq. (12.60), the current in the neutral line is

$$\begin{aligned}\mathbf{I}_n &= -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = -(6.67 - 0.54 + j8.92 + 3.93 - j9.2)\\ &= -10.06 + j0.28 = 10.06\underline{/178.4^\circ}\text{ A}\end{aligned}$$

Solution:

Part of the problem is already solved in Example 12.9. Assume that the wattmeters are properly connected as in Fig. 12.36.

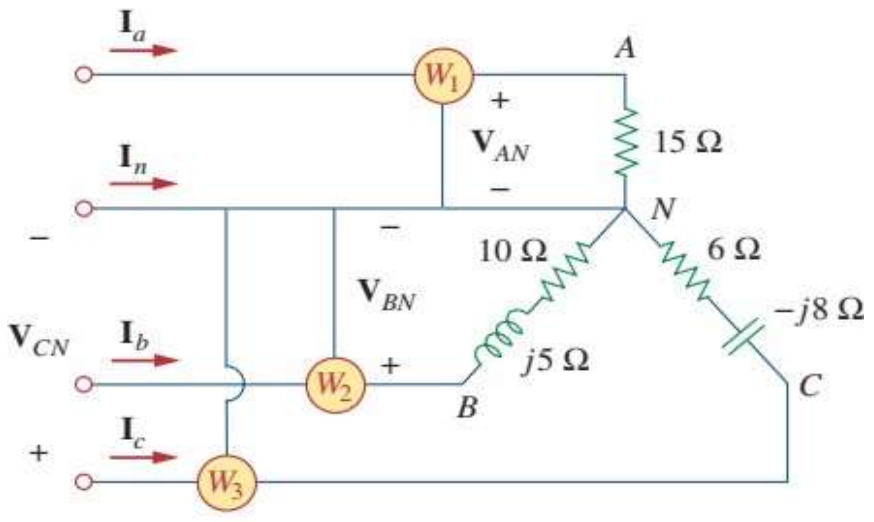

Figure 12.36
For Example 12.13.

(a) From Example 12.9,

$$\mathbf{V}_{AN} = 100\underline{/0^\circ}, \quad \mathbf{V}_{BN} = 100\underline{/120^\circ}, \quad \mathbf{V}_{CN} = 100\underline{/-120^\circ} \text{ V}$$

while

$$\mathbf{I}_a = 6.67\underline{/0^\circ}, \quad \mathbf{I}_b = 8.94\underline{/93.44^\circ}, \quad \mathbf{I}_c = 10\underline{/-66.87^\circ} \text{ A}$$

We calculate the wattmeter readings as follows:

$$\begin{aligned}
P_1 &= \text{Re}(\mathbf{V}_{AN}\mathbf{I}_a^*) = V_{AN}I_a \cos(\theta_{\mathbf{V}_{AN}} - \theta_{\mathbf{I}_a}) \\
&= 100 \times 6.67 \times \cos(0^\circ - 0^\circ) = 667 \text{ W} \\
P_2 &= \text{Re}(\mathbf{V}_{BN}\mathbf{I}_b^*) = V_{BN}I_b \cos(\theta_{\mathbf{V}_{BN}} - \theta_{\mathbf{I}_b}) \\
&= 100 \times 8.94 \times \cos(120^\circ - 93.44^\circ) = 800 \text{ W} \\
P_3 &= \text{Re}(\mathbf{V}_{CN}\mathbf{I}_c^*) = V_{CN}I_c \cos(\theta_{\mathbf{V}_{CN}} - \theta_{\mathbf{I}_c}) \\
&= 100 \times 10 \times \cos(-120^\circ + 66.87^\circ) = 600 \text{ W}
\end{aligned}$$

(b) The total power absorbed is

$$P_T = P_1 + P_2 + P_3 = 667 + 800 + 600 = 2067 \text{ W}$$

We can find the power absorbed by the resistors in Fig. 12.36 and use that to check or confirm this result

$$\begin{aligned}
P_T &= |I_a|^2(15) + |I_b|^2(10) + |I_c|^2(6) \\
&= 6.67^2(15) + 8.94^2(10) + 10^2(6) \\
&= 667 + 800 + 600 = 2067 \text{ W}
\end{aligned}$$

which is exactly the same thing.

Example 12.14

The two-wattmeter method produces wattmeter readings $P_1 = 1560$ W and $P_2 = 2100$ W when connected to a delta-connected load. If the line voltage is 220 V, calculate: (a) the per-phase average power, (b) the per-phase reactive power, (c) the power factor, and (d) the phase impedance.

Solution:

We can apply the given results to the delta-connected load.

(a) The total real or average power is

$$P_T = P_1 + P_2 = 1560 + 2100 = 3660 \text{ W}$$

The per-phase average power is then

$$P_p = \frac{1}{3}P_T = 1220 \text{ W}$$

(b) The total reactive power is

$$Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(2100 - 1560) = 935.3 \text{ VAR}$$

so that the per-phase reactive power is

$$Q_p = \frac{1}{3}Q_T = 311.77 \text{ VAR}$$

(c) The power angle is

$$\theta = \tan^{-1}\frac{Q_T}{P_T} = \tan^{-1}\frac{935.3}{3660} = 14.33°$$

Hence, the power factor is

$$\cos\theta = 0.9689 \text{ (lagging)}$$

It is a lagging pf because Q_T is positive or $P_2 > P_1$.

(c) The phase impedance is $\mathbf{Z}_p = Z_p\underline{/\theta}$. We know that θ is the same as the pf angle; that is, $\theta = 14.33°$.

We recall that for a delta-connected load, $V_p = V_L = 220$ V. From Eq. (12.46),

$$P_p = V_p I_p \cos\theta \quad \Rightarrow \quad I_p = \frac{1220}{220 \times 0.9689} = 5.723 \text{ A}$$

Hence,

$$Z_p = \frac{V_p}{I_p} = \frac{220}{5.723} = 38.44 \ \Omega$$

and

$$\mathbf{Z}_p = 38.44\underline{/14.33°} \ \Omega$$

Example 12.15

The three-phase balanced load in Fig. 12.35 has impedance per phase of $\mathbf{Z}_Y = 8 + j6\ \Omega$. If the load is connected to 208-V lines, predict the readings of the wattmeters W_1 and W_2. Find P_T and Q_T.

Solution:

The impedance per phase is

$$\mathbf{Z}_Y = 8 + j6 = 10\underline{/36.87^\circ}\ \Omega$$

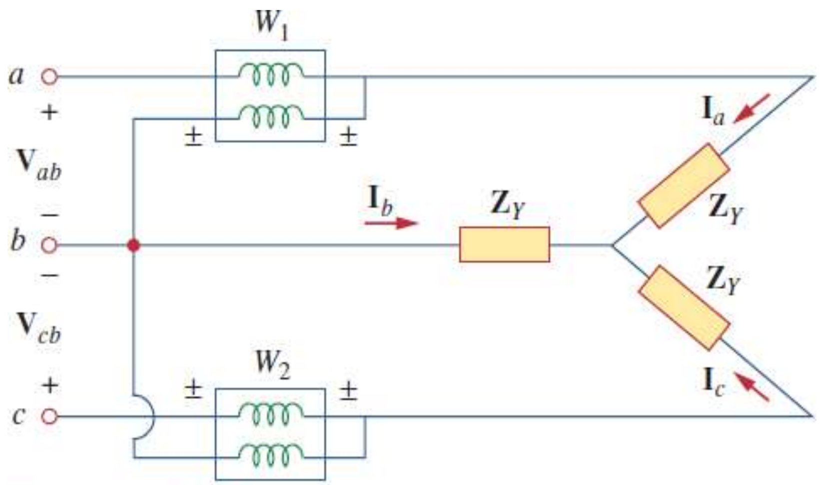

Figure 12.35

so that the pf angle is 36.87°. Since the line voltage $V_L = 208$ V, the line current is

$$I_L = \frac{V_p}{|\mathbf{Z}_Y|} = \frac{208/\sqrt{3}}{10} = 12 \text{ A}$$

Then

$$\begin{aligned} P_1 &= V_L I_L \cos(\theta + 30^\circ) = 208 \times 12 \times \cos(36.87^\circ + 30^\circ) \\ &= 980.48 \text{ W} \\ P_2 &= V_L I_L \cos(\theta - 30^\circ) = 208 \times 12 \times \cos(36.87^\circ - 30^\circ) \\ &= 2478.1 \text{ W} \end{aligned}$$

Thus, wattmeter 1 reads 980.48 W, while wattmeter 2 reads 2478.1 W. Since $P_2 > P_1$, the load is inductive. This is evident from the load $\mathbf{Z}_Y$ itself. Next,

$$P_T = P_1 + P_2 = 3.459 \text{ kW}$$

and

$$Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(1497.6) \text{ VAR} = 2.594 \text{ kVAR}$$

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Effective power, average power, Power triangle

The instantaneous power (in watts) is the power at any instant of time.

$$\boxed{p(t) = v(t)i(t)}$$

The average power, in watts, is the average of the instantaneous power over one period.

Thus, the average power is given by

$$P = \frac{1}{T}\int_0^T p(t)\,dt \qquad \textbf{(11.6)}$$

A resistive load (R) absorbs power at all times, while a reactive load (L or C) absorbs zero average power.

The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

The apparent power (in VA) is the product of the rms values of voltage and current.

The power factor is the cosine of the phase difference between voltage and current. It is also the cosine of the angle of the load impedance.

Complex power (in VA) is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. As a complex quantity, its real part is real power P and its imaginary part is reactive power Q.

The complex, real, and reactive powers of the sources equal the respective sums of the complex, real, and reactive powers of the individual loads.

The process of increasing the power factor without altering the voltage or current to the original load is known as power factor correction.

Power factor improvement plant

11.8 Power Factor Correction

Most domestic loads (such as washing machines, air conditioners, and refrigerators) and industrial loads (such as induction motors) are inductive and operate at a low lagging power factor. Although the inductive nature of the load cannot be changed, we can increase its power factor.

The process of increasing the power factor without altering the voltage or current to the original load is known as **power factor correction**.

Since most loads are inductive, as shown in Fig. 11.27(a), a load's power factor is improved or corrected by deliberately installing a capacitor in parallel with the load, as shown in Fig. 11.27(b). The effect of adding the capacitor can be illustrated using either the power triangle or the phasor diagram of the currents involved. Figure 11.28 shows the latter, where it is assumed that the circuit in Fig. 11.27(a) has a power factor of $\cos\theta_1$, while the one in Fig. 11.27(b) has a power factor of $\cos\theta_2$. It is evident from Fig. 11.28 that adding the capacitor has caused the phase angle between the supplied voltage and current to reduce from θ_1 to θ_2, thereby increasing the power factor. We also notice from the magnitudes of the vectors in Fig. 11.28 that with the

Power factor improvement plant

same supplied voltage, the circuit in Fig. 11.27(a) draws larger current I_L than the current I drawn by the circuit in Fig. 11.27(b). Power companies charge more for larger currents, because they result in increased power losses (by a squared factor, since $P = I_L^2 R$). Therefore, it is beneficial to both the power company and the consumer that every effort is made to minimize current level or keep the power factor as close to unity as possible. By choosing a suitable size for the capacitor, the current can be made to be completely in phase with the voltage, implying unity power factor.

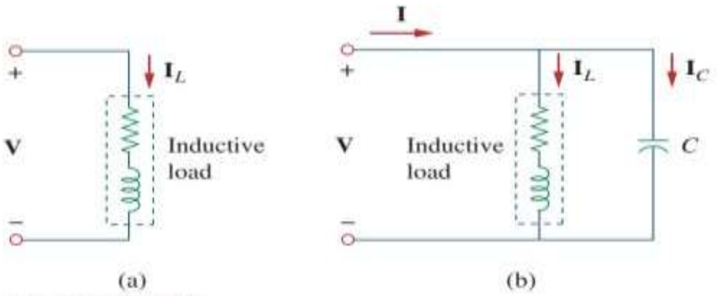

Figure 11.27
Power factor correction: (a) original inductive load, (b) inductive load with improved power factor.

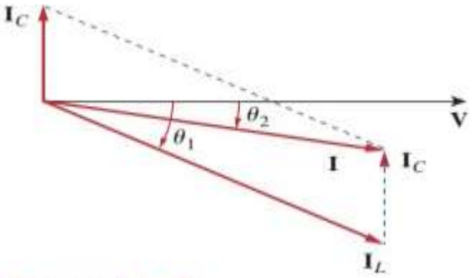

Figure 11.28
Phasor diagram showing the effect of adding a capacitor in parallel with the inductive load.

We can look at the power factor correction from another perspective. Consider the power triangle in Fig. 11.29. If the original inductive load has apparent power S_1, then

$$P = S_1 \cos\theta_1, \qquad Q_1 = S_1 \sin\theta_1 = P \tan\theta_1 \qquad \textbf{(11.57)}$$

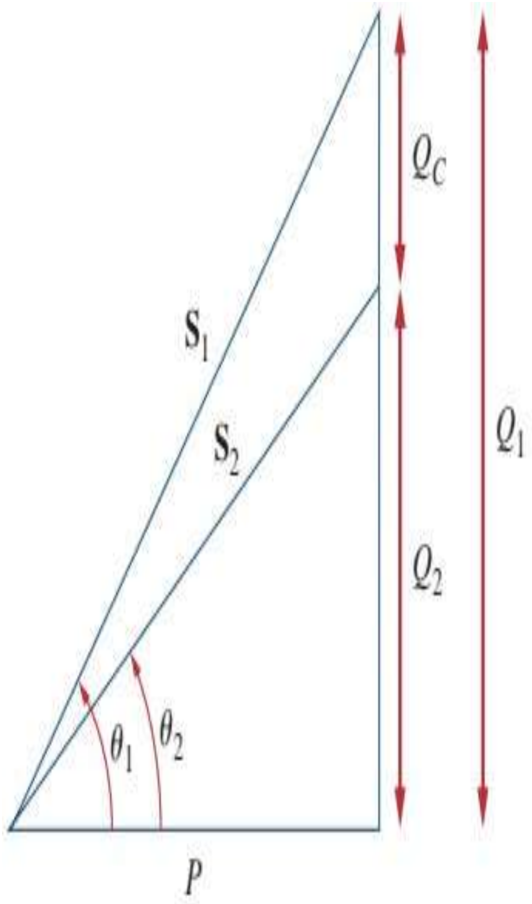

Figure 11.29
Power triangle illustrating power factor correction.

If we desire to increase the power factor from $\cos\theta_1$ to $\cos\theta_2$ without altering the real power (i.e., $P = S_2 \cos\theta_2$), then the new reactive power is

$$Q_2 = P \tan\theta_2 \tag{11.58}$$

The reduction in the reactive power is caused by the shunt capacitor; that is,

$$Q_C = Q_1 - Q_2 = P(\tan\theta_1 - \tan\theta_2) \tag{11.59}$$

But from Eq. (11.46), $Q_C = V_{\text{rms}}^2/X_C = \omega C V_{\text{rms}}^2$. The value of the required shunt capacitance C is determined as

$$\boxed{C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\text{rms}}^2}} \tag{11.60}$$

Note that the real power P dissipated by the load is not affected by the power factor correction because the average power due to the capacitance is zero.

Although the most common situation in practice is that of an inductive load, it is also possible that the load is capacitive; that is, the load is operating at a leading power factor. In this case, an inductor should be connected across the load for power factor correction. The required shunt inductance L can be calculated from

$$Q_L = \frac{V_{\text{rms}}^2}{X_L} = \frac{V_{\text{rms}}^2}{\omega L} \quad \Rightarrow \quad L = \frac{V_{\text{rms}}^2}{\omega Q_L} \tag{11.61}$$

where $Q_L = Q_1 - Q_2$, the difference between the new and old reactive powers.

Example 11.15

When connected to a 120-V (rms), 60-Hz power line, 4 kW at a lagging power factor of 0.8. Find the value necessary to raise the pf to 0.95.

Solution:

If the pf = 0.8, then

$$\cos\theta_1 = 0.8 \qquad \Rightarrow \qquad \theta_1 = 36.87^\circ$$

where θ_1 is the phase difference between voltage ar obtain the apparent power from the real power and the

$$S_1 = \frac{P}{\cos\theta_1} = \frac{4000}{0.8} = 5000 \text{ VA}$$

The reactive power is

$$Q_1 = S_1 \sin\theta = 5000 \sin 36.87 = 3000 \text{ VAR}$$

When the pf is raised to 0.95,

$$\cos\theta_2 = 0.95 \qquad \Rightarrow \qquad \theta_2 = 18.19^\circ$$

The real power P has not changed. But the apparent power has changed; its new value is

$$S_2 = \frac{P}{\cos\theta_2} = \frac{4000}{0.95} = 4210.5 \text{ VA}$$

The new reactive power is

$$Q_2 = S_2 \sin\theta_2 = 1314.4 \text{ VAR}$$

The difference between the new and old reactive powers is due to the parallel addition of the capacitor to the load. The reactive power due to the capacitor is

$$Q_C = Q_1 - Q_2 = 3000 - 1314.4 = 1685.6 \text{ VAR}$$

and

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{1685.6}{2\pi \times 60 \times 120^2} = 310.5\ \mu\text{F}$$

The real power P has not changed. But the apparent power has changed; its new value is

$$S_2 = \frac{P}{\cos\theta_2} = \frac{4000}{0.95} = 4210.5 \text{ VA}$$

The new reactive power is

$$Q_2 = S_2 \sin\theta_2 = 1314.4 \text{ VAR}$$

The difference between the new and old reactive powers is due to the parallel addition of the capacitor to the load. The reactive power due to the capacitor is

$$Q_C = Q_1 - Q_2 = 3000 - 1314.4 = 1685.6 \text{ VAR}$$

and

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{1685.6}{2\pi \times 60 \times 120^2} = 310.5\ \mu\text{F}$$

WEEK-17

Review of all key topics and advanced problem-solving

Course Content	Teaching-Learning Strategy	Sources
Review of all key topics and advanced problem-solving	Revision lectures, practice questions	Course slides and previous sources

Final assessment and student feedback

Course Content	Teaching-Learning Strategy	Sources
Final assessment and student feedback	Comprehensive review and Questions Answers	Course slides and review material

"Learning is not a destination; it is a continuous process."

— Kevin Horsley

Thank YOU!